# UNIT 3

# LINEAR FUNCTIONS

In the *Patterns of Change* unit, you explored a variety of situations in which variables change over time or in response to changes in other variables. Recognizing patterns of change enabled you to make predictions and to understand situations better.

In this unit of *Core-Plus Mathematics*, you will focus on patterns in tables, graphs, and rules of the simplest and one of the most important relationships among variables, linear functions. The understanding and skill needed to analyze and use linear functions will develop from your work on problems in three lessons.

## Lessons

**1 Modeling Linear Relationships**

Identify problem conditions, numeric patterns, and symbolic rules of functions with graphs that are straight lines. Write rules for linear functions given a problem situation or data in a table or a graph. Fit lines and function rules to data patterns that are approximately linear.

**2 Linear Equations and Inequalities**

Express questions about linear functions as equations or inequalities. Use function tables, graphs, and symbolic reasoning to answer those questions.

**3 Equivalent Expressions**

Use context clues and algebraic properties of numbers and operations to recognize and write equivalent forms of symbolic representations of linear functions.
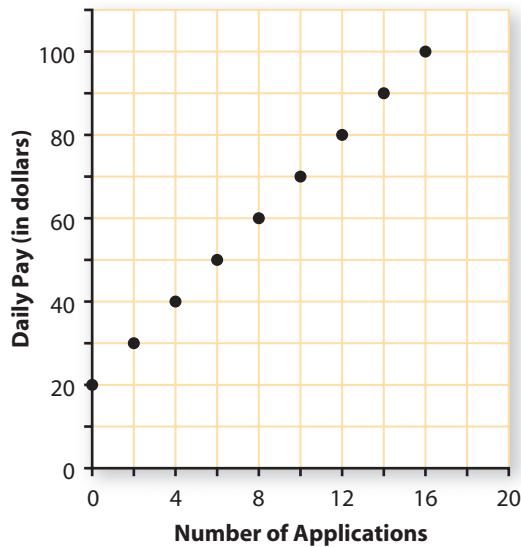
# *Modeling Linear Relationships*

**I**n the *Patterns of Change* unit, you studied a variety of relationships between quantitative variables. Among the most common were **linear functions**—those with straight-line graphs, data patterns showing a constant rate of change in the dependent variable, and rules like $y = a + bx$.

For example, Barry represents a credit card company on college campuses. He entices students with free gifts—hats, water bottles, and T-shirts—to complete a credit card application. The graph on the next page shows the relationship between Barry's daily pay and the number of credit card applications he collects. The graph pattern suggests that *daily pay* is a linear function of *number of applications*.

Jessica Tucker

**Pay for Soliciting Credit Card Customers**

## Think About This Situation

**Think about the connections among graphs, data patterns, function rules, and problem conditions for linear relationships.**

**a)** How does Barry's daily pay change as the number of applications he collects increases? How is that pattern of change shown in the graph?

**b)** If the linear pattern shown by the graph holds for other (*number of applications, daily pay*) pairs, how much would you expect Barry to earn for a day during which he collects just 1 application? For a day he collects 13 applications? For a day he collects 25 applications?

**c)** What information from the graph might you use to write a rule showing how to calculate daily pay for any number of applications?

Working on the problems of this lesson will develop your ability to recognize linear relationships between variables and to represent those relationships with graphs, tables of values, and function rules.

## Investigation 1 — Getting Credit

Information about a linear function may be given in the form of a table or graph, a symbolic rule, or a verbal description that explains how the dependent and independent variables are related. To be proficient in answering questions about linear functions, it helps to be skillful in translating given information from one form into another.

As you work on problems of this investigation, look for clues to help you answer this question:

*How are patterns in tables of values, graphs, symbolic rules, and problem conditions for linear functions related to each other?*

**Selling Credit Cards** Companies that offer credit cards pay the people who collect applications for those cards and the people who contact current cardholders to sell them additional financial services.

1. For collecting credit card applications, Barry's daily pay $B$ is related to the number of applications he collects $n$ by the rule $B = 20 + 5n$.

   **a.** Use the function rule to complete this table of sample $(n, B)$ values:

| Number of Applications | 0 | 1 | 2 | 3 | 4 | 5 | 10 | 20 | 50 |
|---|---|---|---|---|---|---|---|---|---|
| Daily Pay (in dollars) | | | | | | | | | |

   **b.** Compare the pattern of change shown in your table with that shown in the graph on the preceding page.

   **c.** How much will Barry earn on a day when he does not collect any credit card applications? How can this information be seen in the rule $B = 20 + 5n$? In the table of sample $(n, B)$ values? In the graph on the preceding page?

   **d.** How much additional money does Barry earn for each application he collects? How can this information be seen in the rule $B = 20 + 5n$? In the table? In the graph?

   **e.** Use the words *NOW* and *NEXT* to write a rule showing how Barry's daily pay changes with each new credit card application he collects.

2. Cheri also works for the credit card company. She calls existing customers to sell them additional services for their account. The next table shows how much Cheri earns for selling selected numbers of additional services.

| Number of Services Sold | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| Daily Pay (in dollars) | 60 | 80 | 100 | 120 | 140 |

   **a.** Does Cheri's daily pay appear to be a linear function of the number of services sold? Explain.

   **b.** Assume that Cheri's daily pay is a linear function of the number of services she sells, and calculate the missing entries in the next table.

| Number of Services Sold | 0 | 10 | 15 | 20 | 25 | 30 | 40 | 50 | 100 | 101 |
|---|---|---|---|---|---|---|---|---|---|---|
| Daily Earnings (in dollars) | | 60 | | 80 | | 100 | 120 | 140 | | |
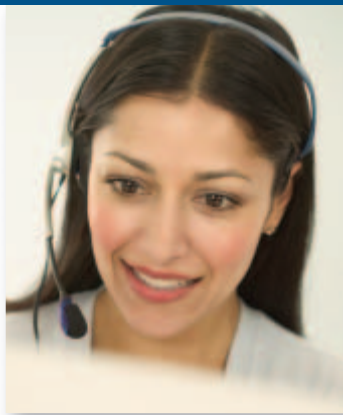
A key feature of any function is the way the value of the dependent variable changes as the value of the independent variable changes. Notice that as the number of services Cheri sells increases from 30 to 40, her pay increases from $100 to $120. This is an increase of $20 in pay for an increase of 10 in the number of services sold, or an average of $2 per sale. Her pay increases at a *rate* of $2 per service sold.

**c.** Using your table from Part b, study the *rate of change* in Cheri's daily pay as the number of services she sells increases by completing entries in a table like the one below.

| Change in Sales | Change in Pay (in $) | Rate of Change (in $ per sale) |
|---|---|---|
| 10 to 20 | | |
| 20 to 25 | | |
| 25 to 40 | | |
| 50 to 100 | | |

What do you notice about the rate of change in Cheri's daily pay as the number of services she sells increases?

**d.** Use the words *NOW* and *NEXT* to write a rule showing how Cheri's pay changes with each new additional service she sells.

**e.** Consider the following function rules.

$$C = 2 + 40n \qquad C = n + 2 \qquad C = 40 + 2n$$
$$C = 50 + \frac{n}{2} \qquad C = 2n + 50$$

   **i.** Which of the rules show how to calculate Cheri's daily pay $C$ for any number of services $n$ she sells? How do you know?

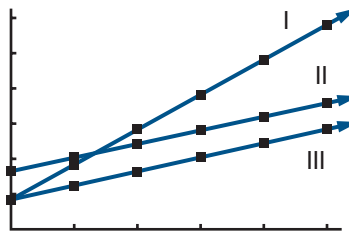   **ii.** What do the numbers in the rule(s) you selected in part i tell you about Cheri's daily pay?

③ The diagram below shows graphs of pay plans offered by three different banks to employees who collect credit card applications.

   Atlantic Bank: $A = 20 + 2n$

   Boston Bank: $B = 20 + 5n$

   Consumers Bank: $C = 40 + 2n$

**a.** Match each function rule with its graph. Explain how you can make the matches without calculations or graphing tool help.



**b.** What do the numbers in the rule for the pay plan at Atlantic Bank tell you about the relationship between daily pay and number of credit card applications collected?

**Buying on Credit** Electric Avenue sells audio/video, computer, and entertainment products. The store offers 0% interest for 12 months on purchases made using an Electric Avenue store credit card.

**4** Emily purchased a television for $480 using an Electric Avenue store credit card. Suppose she pays the minimum monthly payment of $20 each month for the first 12 months.

**a.** Complete a table of (*number of monthly payments, account balance*) values for the first 6 months after the purchase, then plot those values on a graph.

| Number of Monthly Payments | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Account Balance (in dollars) | | | | | | | |

**b.** Will Emily pay off the balance within 12 months? How do you know?

**c.** If you know Emily's account balance *NOW*, how can you calculate the *NEXT* account balance, after a monthly payment?

**d.** Which of the following function rules gives Emily's account balance $E$ after $m$ monthly payments have been made?

$$E = 20m - 480 \qquad E = m - 20 \qquad E = -20m + 480$$
$$E = 480 + 20m \qquad E = 480 - 20m$$

**e.** Determine the rate of change, including units, in the account balance as the number of monthly payments increases from:

> 0 to 2;
>
> 2 to 3;
>
> 3 to 6.

**i.** How does the rate of change reflect the fact that the account balance *decreases* as the number of monthly payments increases?

**ii.** How can the rate of change be seen in the graph from Part a? In the function rule(s) you selected in Part c?

**f.** How can the starting account balance be seen in the table in Part a? In the graph? In the function rule(s) you selected in Part d?

**5** The diagram below shows graphs of account balance functions for three Electric Avenue customers.

> Emily: $E = 480 - 20m$
> Darryl: $D = 480 - 40m$
> Felicia: $F = 360 - 40m$

**a.** Match each function rule with its graph. Explain how you could make the matches without calculations or graphing tool help.

**b.** What do the numbers in the rules for Darryl's and Felicia's account balances tell you about the values of their purchases and their monthly payments?
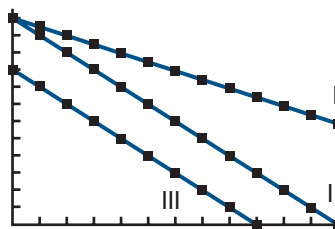
### Linear Functions Without Contexts
When studying linear functions, it helps to think about real contexts. However, the connections among graphs, tables, and symbolic rules are the same for linear functions relating *any* two variables.

You've probably noticed by now that the rate of change of a linear function is constant and that the rate of change corresponds to the direction and steepness of the graph, or the *slope* of the graph.

You can determine the **rate of change** of $y$ as $x$ increases, or the **slope** of the graph between two points, using the ratio:

$$\frac{\text{change in } y}{\text{change in } x} \text{ or } \frac{\Delta y}{\Delta x}.$$
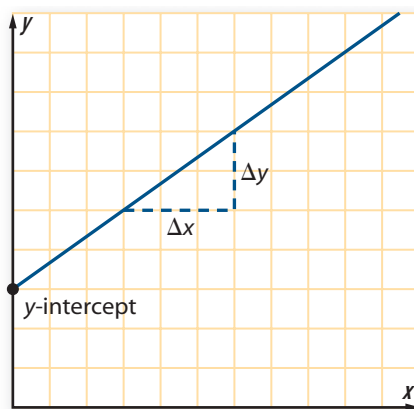
($\Delta$ is the Greek letter "delta," which is used to represent "difference" or "change.")

Another key feature of a linear function is the **y-intercept** of its graph, the point where the graph intersects the $y$-axis.

**6** Draw a graph for each function on a separate set of coordinate axes.

**a.** $y = 1 + \frac{2}{3}x$        **b.** $y = 2x$

**c.** $y = 2x - 3$        **d.** $y = 2 - \frac{1}{2}x$

Then analyze each function rule and its graph as described below.

**i.** Label the coordinates of three points $A$, $B$, and $C$ on each graph. Calculate the slopes of the segments between points $A$ and $B$, between points $B$ and $C$, and between points $A$ and $C$.

**ii.** Label the coordinates of the $y$-intercept on each graph.

**iii.** Explain how the numbers in the symbolic rule relate to the graph.

Linear functions relating two variables *x* and *y* can be represented using tables, graphs, symbolic rules, or verbal descriptions. Key features of a linear function can be seen in each representation.

**a** How can you determine whether a function is linear by inspecting a:

    **i.** table of (*x*, *y*) values?         **ii.** graph of the function?

    **iii.** symbolic rule relating *y* to *x*?     **iv.** *NOW-NEXT* rule?

**b** How can the rate of change and the slope of the graph for a linear function be found from a:

    **i.** table of (*x*, *y*) values?         **ii.** graph of the function?

    **iii.** symbolic rule relating *y* to *x*?     **iv.** *NOW-NEXT* rule?

**c** How can the *y*-intercept of the graph of a function be seen in a:

    **i.** table of (*x*, *y*) values?         **ii.** graph of the function?

    **iii.** symbolic rule relating *y* to *x*?

***Be prepared to share your ideas and reasoning with the class.***
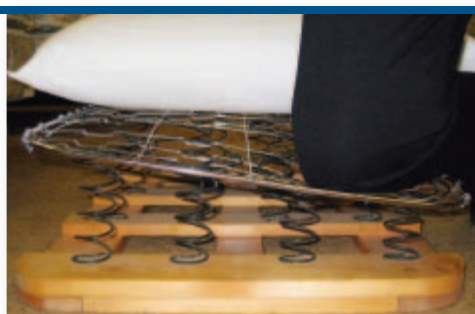
## ✓ Check Your Understanding

Linear functions can be used to describe the action of springs that stretch, like those in telephone cords, and springs that compress, like those in a mattress or a bathroom scale. Hooke's Law in science says that, for an ideal coil spring, the relationship between weight and length is perfectly linear, within the elastic range of the spring.

The table below shows data from an experiment to test Hooke's Law on different coil springs.

| Spring 1 | | Spring 2 | | Spring 3 | | Spring 4 | |
|---|---|---|---|---|---|---|---|
| Weight (ounces) | Length (inches) | Weight (ounces) | Length (inches) | Weight (ounces) | Length (inches) | Weight (ounces) | Length (inches) |
| 0 | 12 | 0 | 5 | 0 | 18 | 0 | 12 |
| 4 | 14 | 2 | 7 | 3 | 15 | 4 | 10 |
| 8 | 16 | 4 | 9 | 6 | 12 | 8 | 8 |
| 12 | 18 | 6 | 11 | 9 | 9 | 12 | 6 |
| 16 | 20 | 8 | 13 | 12 | 6 | 16 | 4 |

For each spring:

**a.** Identify the length of the spring with no weight applied.

**b.** Describe the rate of change of the length of the spring as weight is increased. Indicate units.

**c.** Decide whether the experiment was designed to measure spring stretch or spring compression.

**d.** Write a rule using *NOW* and *NEXT* to show how the spring length changes with each addition of one ounce of weight.

**e.** Match the spring to the rule that gives its length $\ell$ in inches when a weight of $w$ ounces is applied.

$$\ell = 12 - \frac{1}{2}w \qquad \ell = 12 + \frac{1}{2}w \qquad \ell = 5 + w \qquad \ell = 18 - w$$

**f.** Match the spring to the graph in the diagram below that shows $\ell$ as a function of $w$.
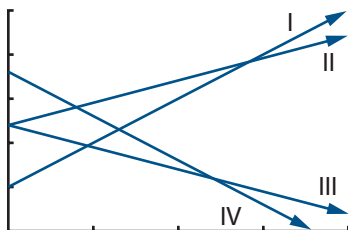


## Investigation 2 Symbolize It

A symbolic rule showing how values of one variable can be calculated from values of another is a concise and simple way to represent a function. Mathematicians typically write the rules for linear functions in the form $y = mx + b$. Statisticians prefer the general form $y = a + bx$. In a linear function rule like $y = 3x + 7$, or equivalently $y = 7 + 3x$, the number 3 is called the **coefficient of $x$** and the number 7 is called the **constant term**.

You probably have several strategies for finding values of the constant term and the coefficient of $x$ in rules for particular linear functions. As you complete the problems in this investigation, look for clues that will help you answer this basic question:

> *How do you use information in a table, a graph, or the conditions of a problem to write a symbolic rule for a linear function?*

**1** **Dunking Booth Profits** The student council at Eastern High School decided to rent a dunking booth for a fund-raiser. They were quite sure that students would pay for chances to hit a target with a ball to dunk a teacher or administrator in a tub of cold water.

The dunking booth costs $150 to rent for the event, and the student council decided to charge students $0.50 per throw.

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**a.** How do you know from the problem description that *profit* is a linear function of the *number of throws*?

**b.** Use the words *NOW* and *NEXT* to write a rule showing how fund-raiser profit changes with each additional customer.

**c.** Write a rule that shows how to calculate the profit $P$ in dollars if $t$ throws are purchased. Explain the thinking you used to write the rule.

**d.** What do the coefficient of $t$ and the constant term in your rule from Part c tell about:

    **i.** the graph of profit as a function of number of throws?

    **ii.** a table of sample (*number of throws, profit*) values?

The description of the dunking booth problem included enough information about the relationship between number of customers and profit to write the profit function. However, in many problems, you will have to reason from patterns in a data table or graph to write a function rule.

**2** **Arcade Prices** Every business has to deal with two important patterns of change, called *depreciation* and *inflation*. When new equipment is purchased, the resale value of that equipment declines or depreciates as time passes. The cost of buying new replacement equipment usually increases due to inflation as time passes.

    The owners of Game Time, Inc. operate a chain of video game arcades. They keep a close eye on prices for new arcade games and the resale value of their existing games. One set of predictions is shown in the graph below.

**a.** Which of the two linear functions predicts the future price of new arcade games? Which predicts the future resale value of arcade games that are purchased now?

### Arcade Prices

Tony Freeman/PhotoEdit

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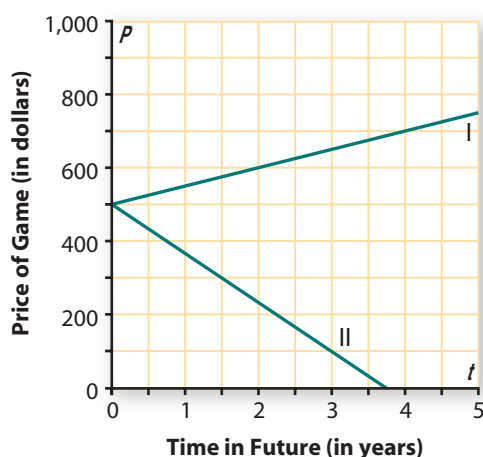
**b.** For each graph:

  **i.** Find the slope and *y*-intercept. Explain what these values tell about arcade game prices.

  **ii.** Write a rule for calculating game price *P* in dollars at any future time *t* in years.

**3** **Turtles** The Terrapin Candy Company sells its specialty—turtles made from pecans, caramel, and chocolate—through orders placed online. The company web page shows a table of prices for sample orders. Each price includes a fixed shipping-and-handling cost plus a cost per box of candy.

| Number of Boxes | 1 | 2 | 3 | 4 | 5 | 10 |
|---|---|---|---|---|---|---|
| Price (in dollars) | 20 | 35 | 50 | 65 | 80 | 155 |

**a.** Explain why that price seems to be a linear function of the number of boxes ordered.

**b.** What is the rate of change in order price as the number of boxes increases?

**c.** Write a rule for calculating the price *P* in dollars for *n* boxes of turtle candies.

**d.** Use your rule to find the price for 6 boxes and the price for 9 boxes of turtle candies.

**4** **Drink Sales** The Washington High School store sells bottled drinks before and after school and during lunch.
   During the first few weeks of school, the store manager set a price of $1.25 per bottle, and daily sales averaged 85 bottles per day. She then increased the price to $1.75 per bottle, and sales decreased to an average of 65 bottles per day.

**a.** What is the rate of change in average daily sales as the price per bottle increases from $1.25 to $1.75? What units would you use to describe this rate of change?

**b.** Assume that sales are a linear function of price. Use the rate of change you found in Part a to reason about expected daily "sales" for a price of $0. Then explain why you would or would not have much confidence in that prediction.

**c.** Use your answers to Parts a and b to write a rule for calculating expected sales *y* for any price *x* in dollars. Check that your rule matches the given information.

**d.** Use your rule to estimate the expected daily sales if the price is set at $0.90 per bottle.

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**5** **Alternate Forms**  It is natural to write rules for many linear functions in **slope-intercept form** like $y = a + bx$ or $y = mx + b$. In some problems, the natural way to write the rule for a linear function leads to somewhat different symbolic forms. It helps to be able to recognize those alternate forms of linear functions. Several rules are given in Parts a–e. For each rule:

   **i.** Decide if it represents a linear function. Explain your reasoning.

   **ii.** If the rule defines a linear function, identify the slope and the $y$-intercept of the function's graph. Write the rule in slope-intercept form.

   **a.** $y = 10 + 2(x - 4)$    **b.** $m = n(n - 5)$

   **c.** $y = 2x^2 - 3$    **d.** $p = (2s + 4) + (3s - 1)$

   **e.** $y = \dfrac{2}{x + 1}$

**6** **Given Two Points**  Each pair of points listed below determines the graph of a linear function. For each pair, give the following.

   **i.** the slope of the graph

   **ii.** the $y$-intercept of the graph

   **iii.** a rule for the function

   **a.** $(0, 5)$ and $(2, 13)$    **b.** $(-3, 12)$ and $(0, 10)$

   **c.** $(-1, 6)$ and $(1, 7)$    **d.** $(3, 9)$ and $(5, 5)$

# Summarize
## the Mathematics

**There are several different methods of writing rules for linear functions.**

**a** To write a rule in the form $y = a + bx$ or $y = mx + b$, how can you use information about:

   **i.** slope and $y$-intercept of the graph of that function?

   **ii.** rate of change and other information in a table of $(x, y)$ values?

**b** How can you determine the rate of change or slope if it's not given directly?

**c** How can you determine the $y$-intercept if it's not given directly?

**d** What is the *NOW-NEXT* rule for a linear function with rule $y = mx + b$? For a function with rule $y = a + bx$?

*Be prepared to share your ideas and reasoning with the class.*

# ✔Check Your Understanding

Write rules in the *NOW-NEXT* and $y = mx + b$ forms for the linear functions that give the following tables and graphs. For the graphs, assume a scale of 1 on each axis.
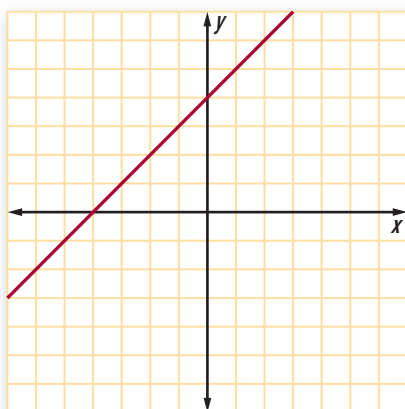
a.

| x | y |
|---|---|
| 5 | 20 |
| 15 | 40 |
| 25 | 60 |
| 35 | 80 |

b.

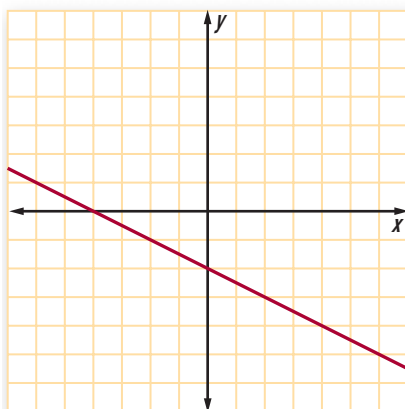| x | y |
|---|---|
| −1 | 8 |
| 0 | 5 |
| 1 | 2 |
| 2 | −1 |

c.


d.


## Investigation 3  Fitting Lines

Linear functions provide useful representations for relationships between variables in many situations, including cases in which data patterns are only approximately linear. As you work on this investigation, look for clues that will help you answer this question:

> *How can you produce and use function rules to represent data patterns that are not perfectly linear?*

**Shadows** On sunny days, every vertical object casts a shadow that is related to its height. The following graph shows data from measurements of flag height and shadow location, taken as a flag was raised up its pole. As the flag was raised higher, the location of its shadow moved farther from the base of the pole.

   Although the points do not all lie on a straight line, the data pattern can be closely approximated by a line.

## Flag Height and Shadow Location
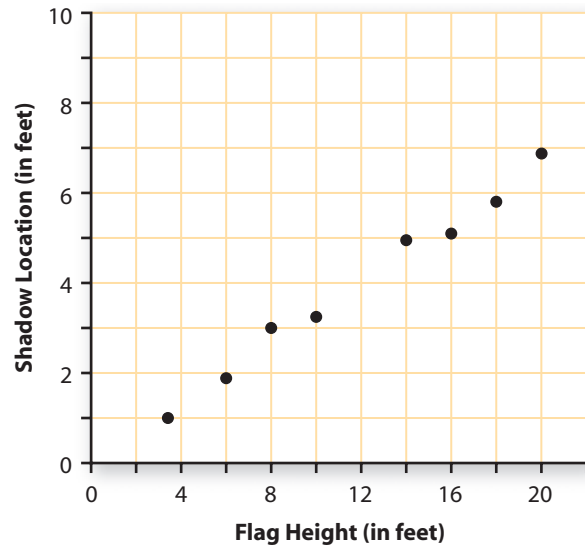


1. Consider the (*flag height, shadow location*) data plotted above.

   **a.** On a copy of the plot, use a straight edge to find a line that fits the data pattern closely. Compare your line with those of your classmates. Discuss reasons for any differences.

   **b.** Write the rule for a function that has your line as its graph.

The line and the rule that match the (*flag height, shadow location*) data pattern are **mathematical models** of the relationship between the two variables. Both the graph and the rule can be used to explore the data pattern and to answer questions about the relationship between flag height and shadow location.

2. Use your mathematical models of the relationship between shadow location and flag height to answer the following questions. Be prepared to explain your strategies for answering the questions.

   **a.** What shadow location would you predict when the flag height is 12 feet?

   **b.** What shadow location would you predict when the flag height is 25 feet?

   **c.** What flag height would locate the flag shadow 6.5 feet from the base of the pole?

   **d.** What flag height would locate the flag shadow 10 feet from the base of the pole?

NASA

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**Time Flies** Airline passengers are always interested in the time a trip will take. Airline companies need to know how flight time is related to flight distance. The following table shows published distance and time data for a sample of United Airlines nonstop flights to and from Chicago, Illinois.

**Nonstop Flights to and from Chicago**

| Travel Between Chicago and: | Distance (in miles) | Flight Time (in minutes) | |
|---|---|---|---|
| | | Westbound | Eastbound |
| Boise, ID | 1,435 | 220 | 190 |
| Boston, MA | 865 | 160 | 140 |
| Cedar Rapids, IA | 195 | 55 | 55 |
| Frankfurt, Germany | 4,335 | 550 | 490 |
| Hong Kong, China | 7,790 | 950 | 850 |
| Las Vegas, NV | 1,510 | 230 | 210 |
| Paris, France | 4,145 | 570 | 500 |
| Pittsburgh, PA | 410 | 95 | 85 |
| San Francisco, CA | 1,845 | 275 | 245 |
| Tokyo, Japan | 6,265 | 790 | 685 |

**Source:** www.uatimetable.com



Scheduled flight time for a given distance depends on many factors, but the factor that has the greatest effect is the speed of prevailing winds. As you can see in the table, westbound flights generally take longer, since the prevailing wind patterns blow from west to east. Therefore, it makes sense to consider westbound flights and eastbound flights separately.

**CPMP-Tools**

**3** To analyze the relationship between westbound flight time and flight distance, study the following scatterplot of the data on westbound flight distance and flight time.

**Westbound Flight Distance and Time**
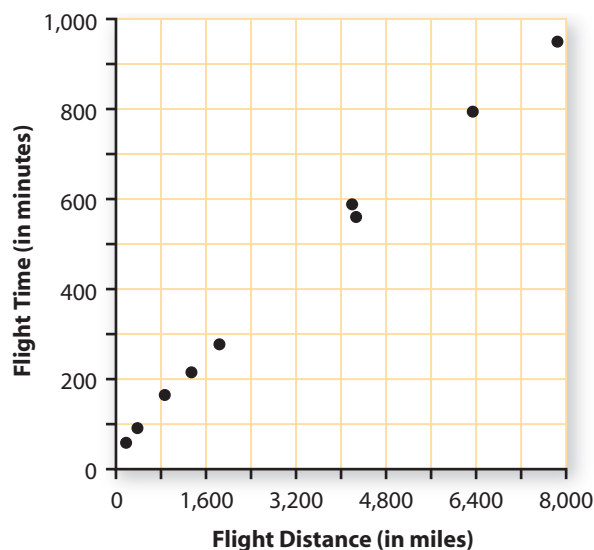


**a.** On a copy of the plot, locate a line that you believe is a good model for the trend in the data. You might find a good modeling line by experimenting with a transparent ruler and pencil. Alternatively, if you have access to software like the Custom Tool "Modeling Line" in *StatTools*, you can manipulate a moveable line on a scatterplot of the data.

When you've located a good modeling line, write a rule for the function that has that line as its graph, using $d$ for distance and $t$ for time.

**b.** Explain what the coefficient of $d$ and the constant term in the rule tell about the relationship between flight time and flight distance for westbound United Airlines flights.

**4** Linear models are often used to summarize patterns in data. They are also useful in making predictions. In the analysis of connections between flight time and distance, this means predicting $t$ from $d$ when no $(d, t)$ pair in the data set gives the answer.

**a.** United Airlines has several daily nonstop flights between Chicago and Salt Lake City, Utah—a distance of 1,247 miles. Use your linear model from Problem 2 to predict the flight time for such westbound flights.

**b.** The scheduled flight times for Chicago to Salt Lake City flights range from 3 hours and 17 minutes to 3 hours and 33 minutes. Compare these times to the prediction of your linear model. Explain why there might be differences between the predicted and scheduled times.

**How's the Weather Up There?** Linear functions are also useful for modeling patterns in climate data. You may have noticed that mountain tops can remain snow-covered long after the snow has melted in the areas below. This is because, in general, the higher you go above sea level, the colder it gets.

Extreme adventurers, such as those who attempt to climb Mt. Everest or those who jump from planes at high altitudes, must protect themselves from harsh temperatures as well as from the lack of oxygen at high altitudes. As skydiver Michael Wright explains about skydiving from 30,000 feet, "Cool? Yes it is. Cold? You bet. Typically 25 below zero (don't be concerned if that is °F or °C, it's still cold)."

An airplane descending to Los Angeles International Airport might record data showing a pattern like that in the next table.

**Airplane Altitude and Temperature Data Above Los Angeles**

| Altitude (in 1,000s of feet) | Temperature (in °F) | Altitude (in 1,000s of feet) | Temperature (in °F) |
|---|---|---|---|
| 34.6 | −58 | 6.6 | 39 |
| 27.3 | −35 | 4.2 | 49 |
| 20.5 | −14 | 2.1 | 57 |
| 13.0 | 13 | 0.6 | 63 |
| 9.5 | 27 | 0.1 | 65 |

**5** When working with paired data, it is helpful to use the list operations provided by calculators and statistical software. To get started, you need to enter the altitude data in one list and the temperature data in another list. Select an appropriate viewing window and produce a plot of the (*altitude, temperature*) data.

**a.** Describe the overall pattern of change in temperature as altitude increases.

**b.** Use two data points to estimate the rate of change in temperature as altitude (in thousands of feet) increases.

**c.** Use the data to make a reasonable estimate of the temperature at an altitude of 0 feet. Then use this value, together with the estimated rate of change from Part b, to write a rule for calculating temperature $T$ as a function of altitude $x$ (in thousands of feet).

**d.** Graph the function from Part c on a scatterplot of the data. Adjust the constant term and the coefficient of $x$ in your rule until you believe the graph of your function closely matches the pattern in the data. Explain how you decided when the line was a good fit.

**e.** The highest elevation in Los Angeles is 5,080 feet at Elsie Peak. Use your linear model from Part d to predict the temperature at Elsie Peak on the day that the other data were collected.

CONTENTS

Linear regression is a branch of statistics that helps in studying relationships between variables. It uses a mathematical algorithm to fit linear models to scatterplot patterns. You will learn more about the algorithm in the Course 2 unit on *Regression and Correlation*. But the algorithm is programmed in most graphing calculators and statistical software for computers, so you can put it to use in mathematical modeling right now.

**6** Use the linear regression algorithm available on your calculator or computer to find a linear function that models the pattern in the (*altitude, temperature*) data, rounding the coefficient of $x$ and the constant term to the nearest tenth.

   **a.** Display the graph of this function on your data plot and compare its fit to that of the function you obtained in Problem 5, Part d.

   **b.** What do the coefficient of $x$ and the constant term in the linear regression rule tell you about the relationship between altitude and temperature?

**7** Skydivers typically jump from altitudes of 10,000 to 15,000 feet. However, high altitude jumping, from 24,000 to 30,000 feet, is becoming popular. Use your linear regression model from Problem 5 to study the temperatures experienced by skydivers at different altitudes on the day the data were collected.

   **a.** Estimate the temperature at altitudes of 10,000, 15,000, 24,000, and 30,000 feet.

   **b.** What change in temperature can be expected as altitude decreases from 30,000 to 24,000 feet? From 24,000 to 15,000 feet? From 15,000 to 10,000 feet? What is the rate of change in temperature as altitude changes in each situation?

   **c.** Frostbite does not occur at temperatures above 28° Fahrenheit. Estimate the altitudes at which temperature is predicted to be at least 28°F.

   **d.** The current world record for skydiving altitude is 102,800 feet, set by Joe Kittinger Jr. in 1960. What temperature does your model predict for an altitude of 102,800 feet? What does this prediction suggest about limits on the linear model for predicting temperature from altitude?

**8** Look back at the scatterplot of United Airlines westbound flight distances and times on page 164. The linear regression model for westbound flight time as a function of flight distance to and from Chicago is approximately $t = 0.12d + 52$.

   **a.** What do the coefficient of $d$ and the constant term in this rule tell you about the relationship between westbound flight time and flight distance?

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Jump Run Productions/Getty Images

**b.** Display the graph of the linear regression model on the scatterplot of (*flight distance, flight time*) data. Compare its fit to that of the modeling function you developed in Problem 3 Part a.

**c.** Describing the rate of change in flight time as flight distance increases in terms of hundredths of a minute per mile is not very informative.

    **i.** Rewrite the rate of change 0.12 as a fraction and explain what the numerator and the denominator of this fraction tell about the relationship between flight time and flight distance.

    **ii.** Express the rate of change in flight time as flight distance increases in terms of minutes per 100 miles.

    **iii.** Write the fraction from part i in an equivalent form that shows the rate of change in minutes per 500 miles. Explain what this fraction suggests about the average speed of westbound planes.

## Summarize the Mathematics

In this investigation, you analyzed tables and plots of sample (*x*, *y*) data to find linear functions that model relationships between variables.

**a** How do you decide if a line is a good fit for the pattern in a data set?

**b** How do you find rules for linear functions that model data patterns that are approximately linear?

**c** How do you use a linear model to estimate *y* values related to given *x* values? To estimate the *x* values that will predict any given *y* values?

*Be prepared to share your thinking and methods with the entire class.*

## ✔ Check Your Understanding

Look back at the United Airlines eastbound flight data on page 15. Since these flights have a tailwind as opposed to a headwind, they take less time.

**a.** On a plot of the eastbound (*flight distance, flight time*) data, locate a line that you believe is a good fit for the pattern in the data.

**b.** What linear function has the line you located in Part a as its graph? What do the coefficient of the independent variable and the constant term in the rule for that function tell about flight distance and flight time for eastbound flights?

**c.** United Airlines has nonstop flights between Chicago and Portland, Maine—a distance of 898 miles. Use your linear model to predict the time for an eastbound flight of 898 miles.

**d.** The scheduled flight times for the Chicago to Portland flights range from 2 hours and 10 minutes to 2 hours and 31 minutes. If your prediction was not in that range, what factors might explain the error of prediction?

# *On Your Own*

## *Applications*

**1** Lake Aid is an annual benefit talent show produced by the students of Wilde Lake High School to raise money for the local food bank. Several functions that relate to Lake Aid finances are described in Parts a–c. For each function:

    **i.** Explain what the numbers in the function rule tell about the situation.

    **ii.** Explain what the function rule tells you to expect in tables of values for the function.

    **iii.** Explain what the function rule tells you to expect in a graph of the function.

    **iv.** Write a *NOW-NEXT* rule to describe the pattern of change in the dependent variable.

  **a.** Several of the show organizers researched the possibility of selling DVDs of the show to increase donations to the food bank. They would have to pay for recording of the show and for production of the DVDs. The cost $C$ (in dollars) would depend on the number of DVDs ordered $n$ according to the rule $C = 150 + 2n$.

  **b.** Proceeds from ticket sales, after security and equipment rental fees are paid, are donated to the local food bank. Once the ticket price was set, organizers determined that the proceeds $P$ (in dollars) would depend on the number of tickets sold $t$ according to the rule $P = 6t - 400$.

  **c.** The organizers of the event surveyed students to see how ticket price would affect the number of tickets sold. The results of the survey showed that the number of tickets sold $T$ could be predicted from the ticket price $p$ (in dollars) using the rule $T = 950 - 75p$.

**2** Given below are five functions and at the right five graphs. Without doing any calculating or graphing yourself, match each function with the graph that most likely represents it. In each case, explain the clues that helped you make the match.



  **a.** $y = x$

  **b.** $y = 2x + 2$

  **c.** $y = 0.1x^2$

  **d.** $y = x + 2$

  **e.** $y = 9 - 0.5x$
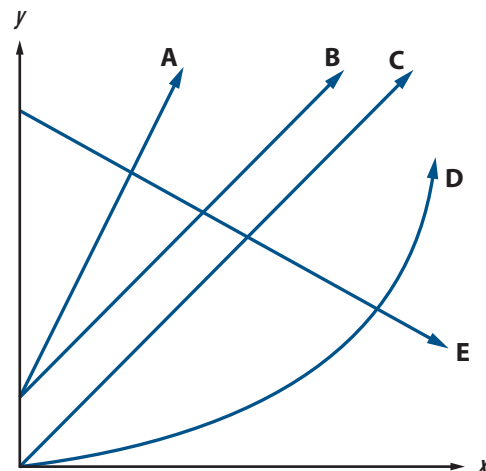
3 The graph below shows the relationship between weekly profit and the number of customers per week for Skate World Roller Rink.

**Skate World Weekly Profit**



Number of Customers per Week
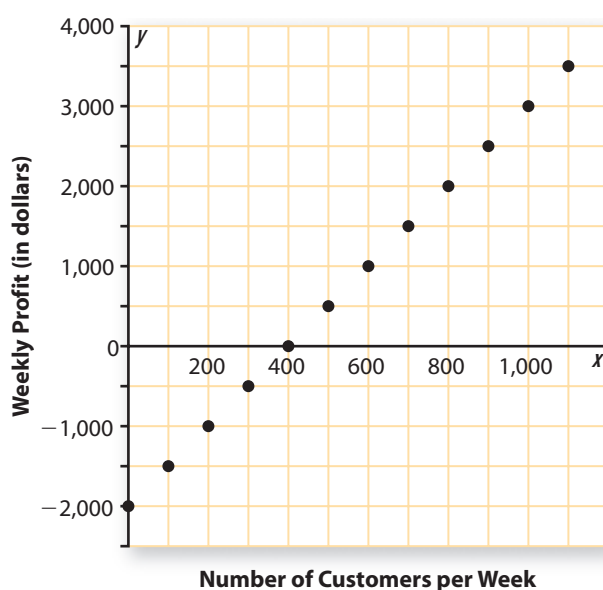
a. Determine the slope and *y*-intercept of the line that fits this data pattern.

b. Explain what the slope and *y*-intercept of the line tell you about the relationship between Skate World profit and number of customers per week.

c. If Skate World reached maximum capacity during each skating session for a week, admissions for that week would total 2,400 customers. Estimate the rink's profit in this situation. Explain your reasoning.

4 The table below gives the amount of money spent on national health care for every ten years from 1960 to 2000.

**U.S. Health-Care Expenditures, 1960–2000**
(in billions of dollars)

| 1960 | 1970 | 1980 | 1990 | 2000 |
|------|------|------|------|------|
| 26.7 | 73.1 | 245.8 | 696.0 | 1,299.5 |

**Source:** *The World Almanac and Book of Facts 2003.* New York, NY: World Almanac Education Group, Inc. 2003.

a. Was the amount of money spent on national health care a linear function over time from 1960 to 2000? Explain how you could tell without plotting the data.

b. What is the rate of change in health-care expenditures from 1960 to 1970? From 1970 to 1980? From 1980 to 1990? From 1990 to 2000? From 1960 to 2000? What does this suggest about the probable shape of a plot of the data?

**5** Victoria got a job at her school as scorekeeper for a summer basketball league. The job pays $450 for the summer and the league plays on 25 nights. Some nights Victoria will have to get a substitute for her job and give her pay for that night to the substitute.

**a.** What should Victoria pay a substitute for one night?

**b.** Use the letters $n$ for nights a substitute works, $S$ for pay to the substitute, and $E$ for Victoria's total summer earnings.

**i.** Write a rule for calculating $S$ as a function of $n$.

**ii.** Write a rule for calculating $E$ as a function of $n$.

**c.** Sketch graphs of the functions that relate total substitute pay and Victoria's total summer earnings to the number of nights a substitute works. Compare the patterns in the two graphs.

**6** Some of the best vacuum cleaners are only sold door-to-door. The salespeople demonstrate the cleaning ability of the appliance in people's homes to encourage them to make the purchase. Michael sells vacuum cleaners door-to-door. He earns a base salary plus a commission on each sale. His weekly earnings depend on the number of vacuum cleaners he sells as shown in the table below.

**Michael's Earnings**

| Number of Vacuum Cleaners Sold in a Week | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| Weekly Earnings (in dollars) | 600 | 960 | 1,320 | 1,680 |

**a.** Verify that weekly earnings are a linear function of the number of vacuum cleaners sold.

**b.** Determine the rate of change in earnings as sales increase. What part of Michael's pay does this figure represent?

**c.** What would Michael's earnings be for a week in which he sold 0 vacuum cleaners?

**d.** Use your answers to Parts b and c to write a rule that shows how Michael's weekly earnings $E$ can be calculated from the number of vacuum cleaners sold in a week $S$.

**e.** Company recruiters claim that salespeople sell as many as 15 vacuum cleaners in a week. What are the weekly earnings for selling 15 vacuum cleaners?

**7** The table below shows the pattern of growth for one bean plant grown under special lighting.

| Day | Height (in cm) |
|---|---|
| 3 | 4.2 |
| 4 | 4.7 |
| 5 | 5.1 |
| 7 | 6.0 |

**a.** Plot the (*day, height*) data and draw a line that is a good fit for the trend in the data.

**b.** Write a function rule for your linear model. What do the numbers in the rule tell about days of growth and height of the bean plant?

**c.** Predict the height of the plant on day 6 and check to see if that prediction seems to fit the pattern in the data table.

**8** For each of these function rules, explain what the constant term and the coefficient of the independent variable tell about the tables and graphs of the function.

**a.** $y = -4 + 2x$

**b.** $p = 7.3n + 12.5$

**c.** $y = 200 - 25x$

**d.** $d = -9.8t + 32$

**9** Write rules for linear functions with graphs containing the following pairs of points.

**a.** (0, 3) and (6, 6)

**b.** (0, −4) and (5, 6)

**c.** (−4, −3) and (2, 3)

**d.** (−6, 4) and (3, −8)

**10** The Riverdale Adventure Club is planning a spring skydiving lesson and first jump. Through the club newsletter, club members were asked to take a poll as to whether or not they would purchase a video of their jump for various prices.

The results of the poll are shown in the table below.

| Cost (in dollars) | 25 | 30 | 35 | 40 | 50 | 60 | 75 |
|---|---|---|---|---|---|---|---|
| Number of Buyers | 93 | 89 | 77 | 71 | 64 | 55 | 38 |

**a.** Create a linear model for the (*cost, number of buyers*) data. Represent your linear model as a graph and as a function rule.

**b.** Use your linear model from Part a to predict the number of members who would purchase a video of their jump for $45. For $70. For $90. For $10. Which estimates would you most trust? Why?

**c.** Should you use your model to predict the number of buyers if videos cost $125? Why or why not?

**d.** For what cost of a video would you predict 50 buyers? 75 buyers? 100 buyers?
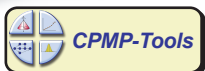
**11** The snowy tree cricket is known as the "thermometer cricket" because it is possible to count its chirping rate and then estimate the temperature. The table below shows the rate of cricket chirps at various temperatures.

| Temperature (in °F) | 50 | 54 | 58 | 61 | 66 | 70 | 75 | 78 | 83 | 86 |
|---|---|---|---|---|---|---|---|---|---|---|
| Chirps per Minute | 41 | 57 | 78 | 90 | 104 | 120 | 144 | 160 | 178 | 192 |

**a.** Use a calculator or computer software to determine the linear regression model for chirp rate $C$ as a function of temperature $T$.

**b.** At what rate do you predict crickets will chirp if the temperature is 70°F? 90°F?

**c.** Now find the linear regression model for temperature $T$ as a function of chirp rate $C$.

**d.** Use the linear regression model from Part c to predict the temperature when crickets are chirping at a rate of 150 chirps per minute. At a rate of 10 chirps per minute. Which prediction would you expect to be more accurate? Why?

**e.** Caution must be exercised in using linear regression models to make predictions that go well beyond the data on which the models are based.

**i.** For what range of temperatures would you expect your linear model in Part a to give accurate chirping rate predictions?

**ii.** For what range of chirping rates would you expect your linear model in Part c to give accurate temperature predictions?

**12** Many Americans love to eat fast food, but also worry about weight. Many fast-food restaurants offer "lite" items in addition to their regular menu items. Examine these data about the fat and calorie content of some fast foods.

| Item | Grams of Fat | Calories |
|---|---|---|
| **McDonald's** | | |
| Grilled Chicken Bacon Ranch Salad | 9 | 260 |
| Grilled Chicken Caesar Salad | 6 | 220 |
| McChicken | 16 | 370 |
| **Hardee's** | | |
| Charbroiled Chicken Sandwich | 26 | 590 |
| Regular Roast Beef | 16 | 330 |
| **Wendy's** | | |
| Mandarin Chicken Salad | 2 | 170 |
| Jr. Cheeseburger | 13 | 320 |
| Ultimate Chicken Grill Sandwich | 7 | 360 |
| **Taco Bell** | | |
| Ranchero Chicken Soft Taco, "Fresco Style" | 4 | 170 |
| Grilled Steak Taco, "Fresco Style" | 5 | 170 |

**Source:** www.mcdonalds.com; www.hardees.com; www.wendys.com; www.tacobell.com

**a.** Make a scatterplot of the data relating calories to grams of fat in the menu items shown.

**b.** Draw a modeling line using the points (6, 220) and (16, 370). Write a rule for the corresponding linear function. Explain what the constant term and the coefficient of $x$ in that rule tell about the graph and about the relation between calories and grams of fat.

**c.** Use your calculator or computer software to find the linear regression model for the (*grams of fat, calories*) data in the table. Compare this result to what you found in Part b.

**13** Over the past 40 years, more and more women have taken full-time jobs outside the home. There has been controversy about whether they are being paid fairly. The table below shows the median incomes for men and women employed full-time outside the home from 1970 to 2003. These data do not show pay for comparable jobs, but median pay for all jobs.

**CONTENTS**

**Median Income** (in dollars)

| Year | Men | Women | Year | Men | Women |
|------|------|-------|------|--------|--------|
| 1970 | 6,670 | 2,237 | 1990 | 20,293 | 10,070 |
| 1975 | 8,853 | 3,385 | 1995 | 22,562 | 12,130 |
| 1980 | 12,530 | 4,920 | 2000 | 28,343 | 16,063 |
| 1985 | 16,311 | 7,217 | 2003 | 29,931 | 17,259 |

**Source:** www.census.gov/hhes/income/histinc/p02.html

**a.** What do you believe are the most interesting and important patterns in these data?

**b.** Did women's incomes improve in relation to men's incomes between 1970 and 2003?

**c.** The diagram below shows a plot of the (*years since 1970, median income*) data for women, using 0 for the year 1970. A linear model for the pattern in those data is drawn on the coordinate grid. Write a function rule for this linear model.

**Women's Median Income**
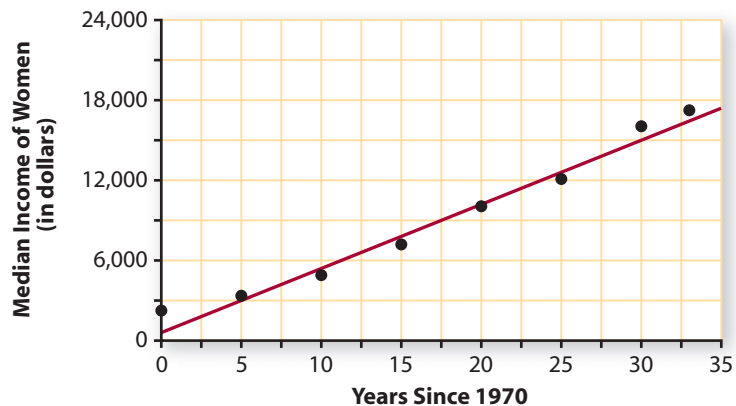


**d.** Using the linear model, estimate the median income of women in 1983 and 2007.

**e.** Is it reasonable to use the model to estimate the median income of women in 1963? Explain.

**f.** Now find the linear regression model for the (*years since 1970, median income*) data. (The data set is in *StatTools*.) Compare the predictions of that model with your results from Part d.

**g.** What do the coefficient and the constant term in the linear model of Part f tell about the pattern of change in median income as time passed between 1970 and 2000?

## *Connections*

**14** Recall the formulas for the circumference of a circle and for the area of a circle.

$$\text{Circumference: } C = 2\pi r \qquad \text{Area: } A = \pi r^2$$

    **a.** Is circumference a linear function of the radius of a circle? Explain how you know.

    **b.** What does the formula for circumference tell about how circumference of a circle changes as the radius increases?

    **c.** Is area a linear function of the radius of a circle? Explain how you know.

    **d.** What does the formula for area tell about how area of a circle changes as the radius of a circle increases?

**15** On hilly roads, you sometimes see signs warning of steep grades ahead. What do you think a sign like the one at the right tells you about the slope of the road ahead?

**16** The diagram at the right shows four linear graphs. For each graph I–IV, do the following.

    **a.** Find the rate at which *y* changes as *x* increases.

    **b.** Write a *NOW-NEXT* rule that describes the pattern of change shown by the graph.

    **c.** Write a rule for calculating *y* as a function of *x*.

    **d.** Explain how your answers relate to each other.
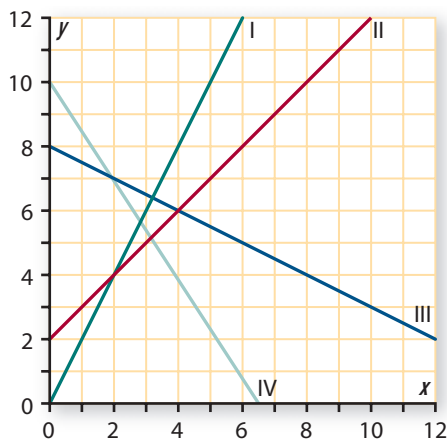
**17** For each table of values (in Parts a and b) use a spreadsheet to reproduce the table.

- Enter the first value for *x* from the table in cell **A1** and the corresponding value for *y* in cell **B1**.

- Plan the spreadsheet so that: (1) the values of *x* appear in column **A**; (2) the corresponding values of *y* are calculated using *NOW-NEXT* reasoning in column **B**; and (3) the corresponding values of *y* are calculated with an appropriate "*y* = ..." formula in column **C**.

- Enter formulas in cells **A2**, **B2**, and **C1** from which the rest of the cell formulas can be generated by application of "fill down" commands.

CONTENTS

**a.**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 7 | 10 | 13 | 16 |

**b.**

| $x$ | 5 | 10 | 15 | 20 |
|---|---|---|---|---|
| $y$ | 30 | 20 | 10 | 0 |

*CPMP-Tools*

**18** The relationship between the temperature measured in degrees Celsius and the temperature measured in degrees Fahrenheit is linear. Water boils at 100°C, or 212°F. Water freezes at 0°C, or 32°F.

**a.** Use this information to write a rule for calculating the temperature in degrees Fahrenheit as a function of the temperature in degrees Celsius.

**b.** Write a rule for calculating the temperature in degrees Celsius as a function of the temperature in degrees Fahrenheit.

**c.** Recall the quote from skydiver Michael Wright on page 16 about skydiving from 30,000 feet, "Cool? Yes it is. Cold? You bet. Typically 25 below zero (don't be concerned if that is °F or °C, it's still cold)."

   **i.** Use your rule from Part a to calculate the equivalent of −25°C in degrees Fahrenheit.

   **ii.** Use your rule from Part b to calculate the equivalent of −25°F in degrees Celsius.

   **iii.** Use a table or graph to determine when it really doesn't matter whether one is talking about °F or °C—when the temperature is the same in both scales.

**19** The table below shows data from the "Taking Chances" investigation (page 9) in Unit 1.

| Number of Trials | 20 | 40 | 60 | 80 | 100 | 120 |
|---|---|---|---|---|---|---|
| Cumulative Profit (in $) | 3 | 7 | 11 | 13 | 15 | 19 |

**a.** Explain why a linear model is reasonable for these data.

**b.** Is cumulative profit an exact linear function of the number of trials? Explain why or why not.

**c.** Use a graphing calculator or computer software to find a linear model for the (*number of trials, cumulative profit*) data.

**d.** What is the coefficient of the independent variable in your model of Part c? What does it tell you about the relationship between cumulative profit and number of trials?

**20** Which of these situations involve linear functions and which do not? Explain your reasoning in each case.

**a.** If a race car averages 150 miles per hour, the distance $d$ covered is a function of driving time $t$.

**b.** If the length of a race is 150 miles, time $t$ to complete the race is a function of average speed $s$.

**c.** If the length of a race is 150 miles, average speed $s$ for the race is a function of race time $t$.

CONTENTS

**21** When Robin and Mike had to find a linear function with graph passing through the two points $A(-3, 12)$ and $Q(4, -2)$, they produced the following work.

The rule will be in the form $y = mx + b$.
The slope of the line is $-2$. (1)
So, $y = -2x + b$. (2)
Since $A(-3, 12)$ is on the line, $12 = -2(-3) + b$ (3)
So, $6 = b$ (4)
So, the rule is $y = -2x + 6$ (5)

**a.** Did Robin and Mike find the correct function rule? If so, what do you think their reasoning was at each step? If not, where did they make an error?

**b.** Use reasoning similar to that of Robin and Mike to find a function rule for the line through the points $(-2, 2)$ and $(6, 10)$.

**c.** Use similar reasoning to find a function rule for the line through the points $(3, 5)$ and $(8, -15)$.

**22** The diagram at the right shows four *parallel lines*.

**a.** For each line I–IV, find its slope and a rule for the function with that graph.

**b.** Write rules for two different linear functions with graphs parallel to the given graphs. Explain how you know that your lines are parallel to the given lines.
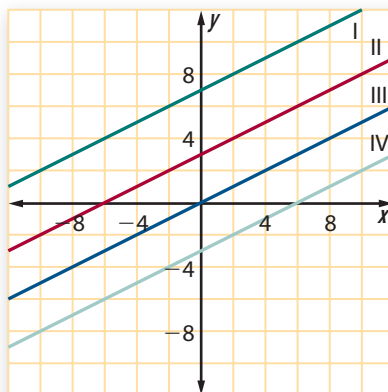


## Reflections

**23** To use linear functions wisely it helps to be in the habit of asking, "What sorts of numbers would make sense in this situation?" For example, in the function relating profit $P$ to the number of customers $n$ at the Starlight Cinema, it would not make much sense to substitute negative values for $n$ in the formula $P = 6.5n - 2,500$. In each of the following situations, decide what range of values for the variables would make sense. It may be helpful to examine tables of values or graphs for some of the functions.

**a.** Suppose a ball is tossed into the air with an upward velocity of 40 feet per second. Its upward velocity is a function of time in flight, according to the formula $V = 40 - 32T$. Velocity $V$ is in feet per second and time $T$ is in seconds. What range of values for $T$ and $V$ make sense in this context?

**b.** The resale value $R$ in dollars of an arcade game is given by $R = 500 - 133T$, where $T$ is time in years after the purchase of the new equipment. What range of values for $R$ and $T$ make sense?

**c.** In one apartment building, new renters are offered $150 off their first month's rent, then they pay a normal rate of $450 per month. The total rent $R$ paid for an apartment in that building is given by $R = 450m - 150$, where $m$ is the number of months. What range of values for $R$ and $m$ make sense?

**24** Think about how the values of the constant term and the coefficient are related to the graphs of linear functions. Suppose you enter the rule $y = 2 + 1.5x$ in your graphing calculator and produce a graph in the standard viewing window.

**a.** How will the graph that you see be different from that of $y = 2 + 1.5x$ if you:

**i.** increase or decrease the coefficient of $x$?

**ii.** increase or decrease the constant term?

**b.** Draw sketches that show possible graphs of functions $y = a + bx$ for each of these cases.

   **i.** $a < 0$ and $b > 0$         **ii.** $a > 0$ and $b < 0$

   **iii.** $a < 0$ and $b < 0$       **iv.** $a > 0$ and $b > 0$

**25** Answer each of the following questions. In each case, explain how the answers can be determined without actually graphing any functions.

**a.** Is the point $(-3, -4)$ on the graph of the line $y = \frac{4}{3}x$?

**b.** Will the graphs of $y = 3x + 7$ and $y = 2 + 3x$ intersect?

**c.** Will the graphs of $y = 3x + 7$ and $y = 2 - 3x$ intersect?

**26** In finding a linear function that models a data pattern, sometimes students simply draw a line connecting two points that are at the left and right ends of the scatterplot. Sketch a scatterplot showing how this simple strategy can produce quite poor models of data patterns.

**27** Investigate the linear regression procedure for finding a linear model to fit data patterns.

**a.** For each of the following data sets, use your calculator or computer software to make a data plot. Then use linear regression to find a linear model and compare the graph produced by the linear regression model to the actual data pattern.

**i.**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |

**ii.**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 29 |

**iii.**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |

**iv.**

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 2 | 5 | 10 | 17 | 26 | 37 | 50 | 65 |

| v. | $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|----|-----|---|---|----|----|----|----|---|---|----|
|    | $y$ | 3 | 8 | 11 | 12 | 12 | 11 | 8 | 3 | −4 |

**b.** What limitations of the linear regression procedure are suggested by the results of your work in Part a?

**28** Consider the information needed to draw a line or find the equation for a line. How many different lines are determined by each of these conditions?

**a.** pass through the points $(−4, 1)$ and $(2, 4)$

**b.** pass through the point $(2, 1)$ and have slope $−2$

**c.** pass through the point $(2, 4)$

**d.** have slope $−2$

**e.** pass through the points $(0, 0)$, $(1, 1)$, and $(2, 3)$

**f.** pass through the points $(0, 1)$, $(1, 2)$, $(2, 3)$, and $(3, 4)$

## *Extensions*

**29** In Connections Task 17, you wrote spreadsheet programs to produce tables of $x$ and $y$ values for two linear functions. You can extend those ideas to produce tables of values for any linear function when given only the starting $x$ and $y$ values and the pattern of change in the $x$ and $y$ values, $\Delta x$ and $\Delta y$.

Complete the spreadsheet program begun below in a way that will allow you to enter specific start and change values and see the corresponding table of $x$ and $y$ values automatically.

- Assume that the starting $x$ value will be entered in cell **E1**, the starting $y$ value in cell **E2**, $\Delta x$ in cell **E3**, and $\Delta y/\Delta x$ in cell **E4**.

- Plan the spreadsheet so that: (1) the values of $x$ appear in column **A**; (2) the corresponding values of $y$ are calculated using *NOW-NEXT* reasoning in column **B**; and (3) the corresponding values of $y$ are calculated with an appropriate "$y = ...$" formula in column **C**.

- Enter formulas in cells **A1**, **A2**, **B1**, **B2**, and **C1** from which the rest of the cell formulas can be generated by application of "fill down" commands.

| Spreadsheet sample.xls | | | | ⬜ 🗗 ❌ |
|---|---|---|---|---|
| ◇ | **A** | **B** | **C** | **D** | **E** |
| **1** | | | | Start $x =$ | |
| **2** | | | | Start $y =$ | |
| **3** | | | | $\Delta x =$ | |
| **4** | | | | $\Delta y/\Delta x =$ | |
| **5** | | | | | |

**30** Carefully graph the line of $y = \frac{2}{3}x + 1$ on grid paper.

a. What is the slope of this line?

b. Using a protractor or mira, carefully draw a line perpendicular to the graph of $y = \frac{2}{3}x + 1$ through the point (0, 1).

c. What is the slope of the perpendicular line? How does the slope of this line compare to the slope you determined in Part a?

d. Write an equation for the perpendicular line.

e. Will all lines having the slope you determined in Part c be perpendicular to all lines having the slope you determined in Part a? Explain why or why not.

f. Carefully draw another pair of perpendicular lines on grid paper and compare their slopes. Explain your conclusions.

**31** The graph below illustrates the relationship between time in flight and height of a soccer ball kicked straight up in the air. The relation is given by $H = -4.9t^2 + 20t$, where $t$ is in seconds and $H$ is in meters.

a. What could it mean to talk about the slope of this curved graph? How could you estimate the slope of the graph at any particular point?

b. How would you measure the rate of change in the height of the ball at any point in its flight?

c. What would rate of change in height or slope of the graph tell about the motion of the ball at any point in its flight?
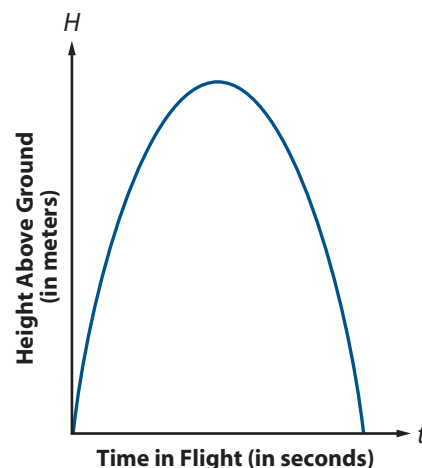


**32** The following scatterplot shows grade point averages of some Wisconsin students in their eighth- and ninth-grade school years. The graph of $y = x$ is drawn on the plot.

a. What is true about the students represented by points that lie on the line $y = x$? That lie above the line $y = x$? That lie below the line $y = x$?

b. The linear regression model for these data is approximately $y = 0.6x + 1.24$. What do the numbers 0.6 and 1.24 tell you about the relationship between eighth- and ninth-grade averages for the sample of students in this study?

CONTENTS
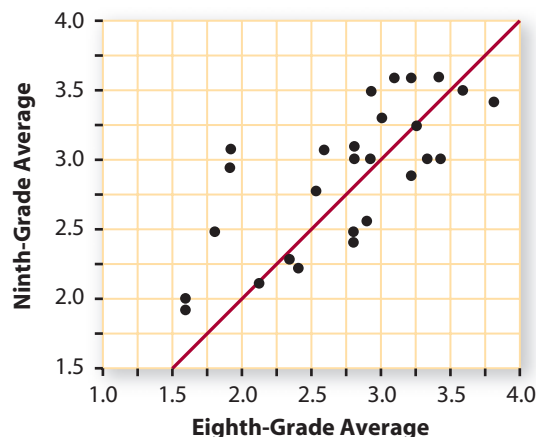
**33** In this lesson, you fitted linear function models to data patterns. You can also use lines to analyze data that do not have a functional relationship. For example, consider the next scatterplot that shows average maximum temperatures in January and July for selected cities around the world.
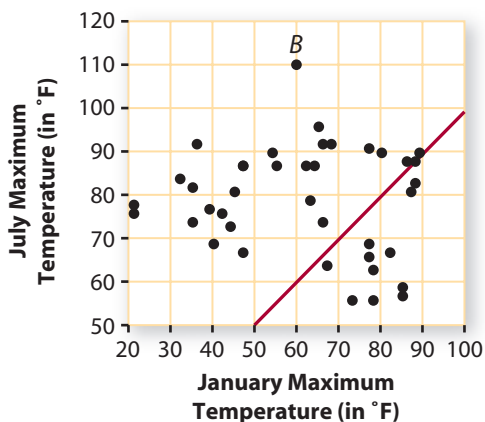


a. What rule describes the line drawn on the scatterplot? If a city is represented by a point on the line, what is true about that city?

b. What is true about the cities represented by points located below the line? Where do you think these cities are located geographically?

c. What is the difference in average temperature in July versus January for the city that is represented by point *B*?

d. Would it be useful to use linear regression on this data set? Explain your reasoning.

**34** The 100-meter run for men has been run in the Olympics since 1896. The winning times for each of the years through 2004 are given in the following table.
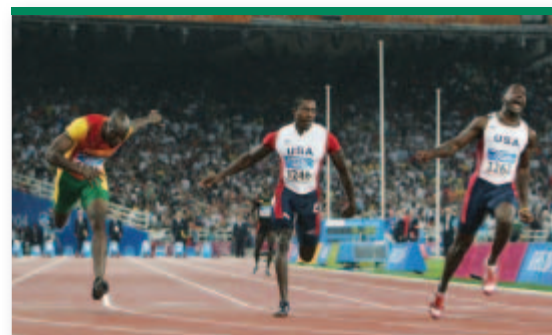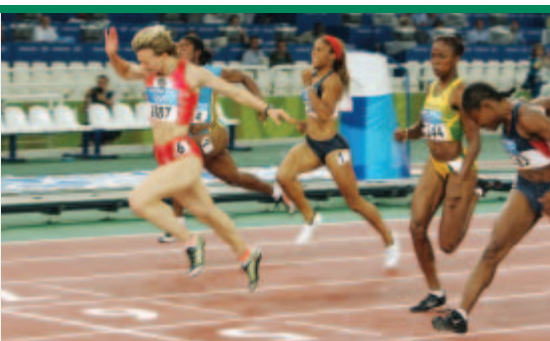
### Winning Times for Men: Olympic 100 Meters

| Year | Time (sec) | Year | Time (sec) | Year | Time (sec) |
|------|-----------|------|-----------|------|-----------|
| 1896 | 12.0 | 1936 | 10.3 | 1980 | 10.25 |
| 1900 | 10.8 | 1948 | 10.3 | 1984 | 9.99 |
| 1904 | 11.0 | 1952 | 10.4 | 1988 | 9.92 |
| 1908 | 10.8 | 1956 | 10.5 | 1992 | 9.96 |
| 1912 | 10.8 | 1960 | 10.2 | 1996 | 9.84 |
| 1920 | 10.8 | 1964 | 10.0 | 2000 | 9.87 |
| 1924 | 10.6 | 1968 | 9.95 | 2004 | 9.85 |
| 1928 | 10.8 | 1972 | 10.14 | | |
| 1932 | 10.3 | 1976 | 10.06 | | |

**Source:** *The World Almanac and Book of Facts 2001*. Mahwah, NJ: World Almanac Education Group, Inc. 2001; www.olympics.com

**a.** There are no 100-meter race times for 1916, 1940, and 1944. Why are these data missing?

**b.** Make a plot of the (*year, time*) data using 1890 as year 0. Then decide whether you think a linear model is reasonable for the pattern in your plot. Explain your reasoning.

**c.** Find a linear model for the data pattern.

**d.** Use your model from Part c to answer the following questions.

   **i.** What winning times would you predict for the 1940 and for the 2008 Olympics?

   **ii.** In what year is the winning time predicted to be 9.80 seconds or less?

   **iii.** In what Olympic year does the model predict a winning time of 10.4 seconds? Compare your prediction to the actual data.

**e.** According to your linear model, by about how much does the men's winning time change from one Olympic year to the next?

**f.** What reasons can you imagine for having doubts about the accuracy of predictions from the linear model for change in winning time as years pass?

**35** Women began running 100-meter Olympic races in 1928. The winning times for women are shown in the table below.

**Winning Times for Women: Olympic 100 Meters**

| Year | Time (sec) | Year | Time (sec) |
|------|------------|------|------------|
| 1928 | 12.2 | 1972 | 11.07 |
| 1932 | 11.9 | 1976 | 11.08 |
| 1936 | 11.5 | 1980 | 11.60 |
| 1948 | 11.9 | 1984 | 10.97 |
| 1952 | 11.5 | 1988 | 10.54 |
| 1956 | 11.5 | 1992 | 10.82 |
| 1960 | 11.0 | 1996 | 10.94 |
| 1964 | 11.4 | 2000 | 10.75 |
| 1968 | 11.0 | 2004 | 10.93 |

**Source:** *The World Almanac and Book of Facts 2001.* Mahwah, NJ: World Almanac Education Group, Inc. 2001; www.olympics.com

**a.** Study the data and describe patterns you see in change of winning race time as years pass.

**b.** Make a plot and then find a linear model for the data pattern. Use 1900 as year 0.

**c.** Use your linear model to answer each of the following questions. For questions ii–iv, compare your predictions to the actual data.

   **i.** What winning time would you predict for 1944?

   **ii.** What winning time does the model predict for 1996?

   **iii.** In what Olympic year does the model suggest there will be a winning time of 10.7 seconds?

   **iv.** According to the model, when should a winning time of 11.2 seconds have occurred?

**d.** According to the model, by about how much does the women's winning time change from one Olympic year to the next? Compare this to your answer for Part e of Extensions Task 34.

## *Review*

**36** Identify all pairs of similar triangles. Then for each pair of similar triangles identify the scale factor.
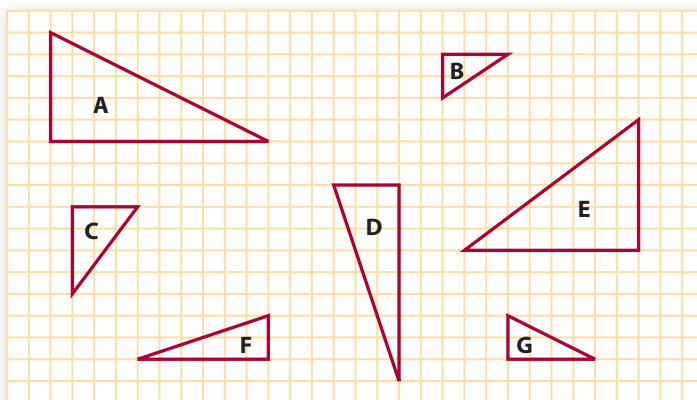


**37** Solve each of the following equations for $x$.

  **a.** $3x = 1$         **b.** $\frac{4}{3}x = 1$         **c.** $4 \div \frac{4}{3} = x$

  **d.** $\frac{4}{5} \div \frac{1}{3} = x$     **e.** $x \div \frac{3}{13} = 1$     **f.** $\frac{1}{5} \div \frac{3}{13} = x$

**38** Translating problem conditions into mathematical statements is an important skill.

**a.** Which of these mathematical statements uses the letters $S$ for number of students and $T$ for number of teachers to express correctly the condition, "At Hickman High School there are 4 student parking places for every teacher parking place"?

$$4S = T \qquad S = 4T \qquad S = T + 4 \qquad T = S + 4$$

**b.** Which of these mathematical statements correctly uses $T$ for tax and $P$ for price to express the fact that, "In California stores, an 8% sales tax is charged on the price of every purchase"?

$$0.8T = P \qquad T = 0.8P \qquad P + 0.08 = T \qquad T = 0.08P$$

**39** Consider the pentagon shown below.
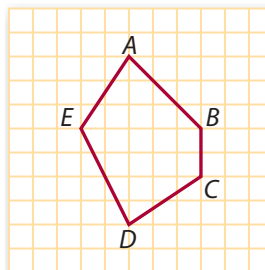


**a.** How many diagonals does this shape have? Name them.

**b.** Do any of the diagonals seem to bisect each other? Explain your reasoning.

**c.** Is $\overline{AD} \perp \overline{BE}$? Explain your reasoning.

**40** Smart supermarket shoppers use unit prices to compare values for products.

**a.** For each of the following comparisons, decide which item is the better buy by finding the unit prices of each.

  **i.** an 18-ounce box of cereal for $3.50, or a 24-ounce box for $4.50?

  **ii.** a 32-ounce jar of spaghetti sauce for $4.25, or a 20-ounce jar for $2.50?

  **iii.** a 6-pack of 20-ounce soft drink bottles for $3.25, or a 12-pack of 12-ounce cans for $4?

**b.** What is the connection between unit prices and slopes or rates of change for linear functions?

**41** Carlos surveyed 120 ninth graders at his school and asked what is their least favorite chore. The results of his survey are provided in the table below.

| Chore | Number of People |
|---|---|
| Cleaning bathroom | 45 |
| Mowing lawn | 30 |
| Walking the dog | 15 |
| Raking leaves | 10 |
| Doing the dishes | 20 |

a. What percentage of the people surveyed said that cleaning the bathroom is their least favorite chore?

b. Make a bar graph of the survey results.

c. Make a circle graph that displays the results of Carlos's survey.

**42** In the figures below, tell whether the orange shape appears to be the reflected image of the green shape across the given line. If it is not, explain how you know.

a.

b.

c.

d.

e.
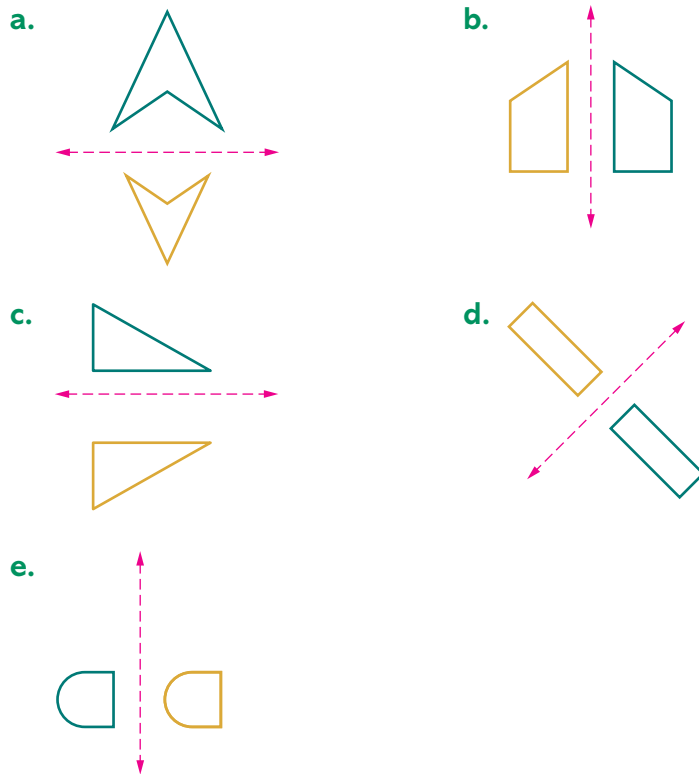
**43** On the first three tests in a marking period, D'Qwell has scores of 85, 90, and 75. To earn a B in the course, he needs a mean test score of at least 85, while an A requires a minimum mean score of 93.

a. What score must D'Qwell get on the final test to earn a B?

b. What score, if any, would earn him an A?

c. If D'Qwell has a quiz average of 7 on the first 9 quizzes of a marking period, how will his quiz average change if he gets a 10 on the next quiz?

# *Linear Equations and Inequalities*

**F**or most of the twentieth century, the vast majority of American medical doctors were men. However, during the past 40 years there has been a significant increase in the number of women graduating from medical schools. As a result, the percent of doctors who are women has grown steadily to nearly 25% in 2000. The graph on the next page shows this trend.

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**Male and Female Medical Doctors**
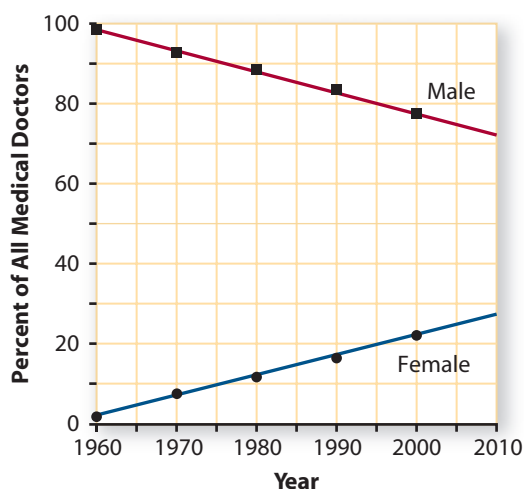


Source: www.ama-assn.org/ama/pub/article/171–195.html

## Think About This Situation

**Study the trends in the percentage of male and female medical doctors in the United States between 1960 and 2000.**

**a** How would you describe the trends shown in the data plots and the linear models that have been drawn to match patterns in those points?

**b** Why do you suppose the percentage of women doctors has been increasing over the past 40 years?

**c** Would you expect the trend in the graph to continue 10 or 20 years beyond 2000?

**d** How would you go about finding function rules to model the data trends?

**e** If you were asked to make a report on future prospects for the percentages of male and female doctors, what kinds of questions could you answer using the linear models?

In this lesson, you will explore ways to express questions about linear functions as equations or inequalities. You will use tables, graphs, and symbolic reasoning to solve those equations and inequalities and to interpret your solutions in problem contexts.

Several kinds of questions occur naturally in thinking about trends in the percentage of male and female medical doctors. To plan for future educational programs and medical services, medical schools, hospitals, and clinics might wonder:

(1) In 2020, what percent of U.S. medical doctors will be female?

(2) When will the percent of female doctors reach 40%?

(3) When will the percent of male and female doctors be equal?

(4) How long will the percent of male doctors remain above 70%?

The trends in percent of male and female medical doctors can be modeled by these linear functions.

**Percentage of Male Doctors:** $y_1 = 98 - 0.54t$
**Percentage of Female Doctors:** $y_2 = 2 + 0.54t$

Here $y_1$ and $y_2$ represent the percentage of male and female U.S. medical doctors at a time $t$ years after 1960. An estimate for the answer to question (1) above can be calculated directly from the function giving percentage of female doctors. Since 2020 is 60 years after 1960, to predict the percent of female doctors in that year we evaluate the expression $2 + 0.54t$ for $t = 60$.

$$y_2 = 2 + 0.54(60), \text{ or } y_2 = 34.4$$

The other three questions above can be answered by solving two algebraic equations and an inequality. In each case, the problem is to find values of $t$ (years since 1960) when the various conditions hold.

$$
\begin{array}{ll}
2 + 0.54t = 40 & (2) \\
98 - 0.54t = 2 + 0.54t & (3) \\
98 - 0.54t > 70 & (4)
\end{array}
$$

As you work on the problems of this investigation, keep in mind the following questions:

*How do you represent questions about linear functions symbolically?*

*How can you use tables and graphs to estimate solutions of equations and inequalities?*

**1** Write equations or inequalities that can be used to estimate answers for each of these questions about the percentage of male and female medical doctors in the United States.

**a.** In 1985, what percent of U.S. medical doctors were male?

**b.** When will the percent of male doctors fall to 40%?

**c.** How long will the percent of female doctors remain below 60%?

**d.** When will the percent of male doctors decline to only double the percent of female doctors?

**2** Write questions about trends in percent of male and female medical doctors that can be answered by solving these equations and inequalities.

**a.** $98 - 0.54t = 65$

**b.** $y_2 = 2 + 0.54(50)$

**c.** $2 + 0.54t < 30$

**d.** $98 - 0.54t > 2 + 0.54t$

**e.** $98 - 0.54t = 4(2 + 0.54t)$

Writing equations and inequalities to match important questions is only the first task in solving the problems they represent. The essential next step is to **solve the equations** or to **solve the inequalities**. That is, find values of the variables that satisfy the conditions.

One way to estimate solutions for equations and inequalities that match questions about percentages of male and female medical doctors is to make and study tables and graphs of the linear models.

**Trends in Gender**
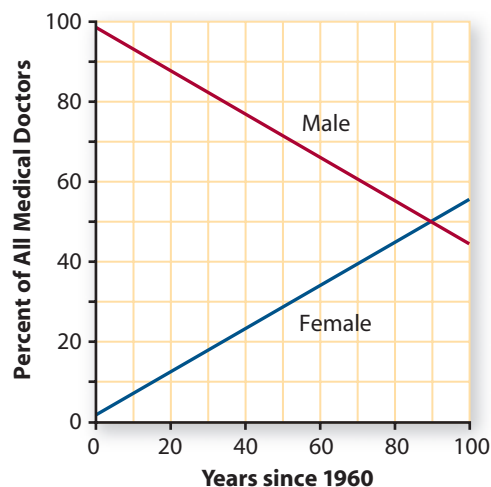


$$y_1 = 98 - 0.54t \qquad \text{and} \qquad y_2 = 2 + 0.54t$$

| t | $y_1$ | $y_2$ |
|---|---|---|
| 0 | 98.0 | 2.0 |
| 10 | 92.6 | 7.4 |
| 20 | 87.2 | 12.8 |
| 30 | 81.8 | 18.2 |
| 40 | 76.4 | 23.6 |

| t | $y_1$ | $y_2$ |
|---|---|---|
| 50 | 71.0 | 29.0 |
| 60 | 65.6 | 34.4 |
| 70 | 60.2 | 39.8 |
| 80 | 54.8 | 45.2 |
| 90 | 49.4 | 50.6 |

**3** For the next equations and inequalities:

- Use the tables and graphs above to estimate the value or range of values that satisfy the given condition.

- Explain what each solution tells about the percentages of male and female medical doctors in the United States.

- Be prepared to explain or show how you used a table or graph to estimate the solution.

**a.** $y_2 = 2 + 0.54(40)$      **b.** $98 - 0.54t = 90$

**c.** $98 - 0.54t = 2 + 0.54t$      **d.** $98 - 0.54t > 80$

**e.** $y_1 = 98 - 0.54(65)$      **f.** $2 + 0.54t < 29$

**g.** $98 - 0.54t = 4(2 + 0.54t)$      **h.** $70 = 2 + 0.54t$

**4** Write equations and inequalities to represent the following questions. Then use tables or graphs to estimate the solutions for the equations and inequalities and explain how those solutions answer the original questions. Be prepared to explain or show how you used a table or graph to estimate the solutions.

  **a.** When will the percent of male doctors decline to 55%?

  **b.** When will the percent of female doctors reach 35%?

  **c.** How long will the percent of male doctors be above 40%?

  **d.** What percent of U.S. medical doctors will be female when you are 20 years old?

  **e.** Assuming the trends shown in the graph on page 187, when will the percent of male doctors be less than the percent of female doctors?

**5** When you solve an equation or inequality, it is always a good idea to check the solution you find.

  **a.** Suppose one person told you that the solution to $45 = 98 - 0.54t$ is $t = 100$, and another person told you that the solution is $t = 98$. How could you check to see if either one is correct without using a table or a graph?

  **b.** How do you know whether a solution is *approximate* or *exact*?

  **c.** If a solution for an equation is exact, does that mean that the answer to the prediction question is certain to be true? Explain.

**6** If someone told you that the solution to $2 + 0.54t \le 45$ is $t \le 80$, how could you check the proposed solution:

  **a.** Using a table?          **b.** From a graph?

  **c.** Using a computer algebra system?

  **d.** Without using a table, a graph, or a computer algebra system?

  **e.** If you wanted to see if a solution is *exact*, which method of checking would you use?

## Summarize
### the Mathematics

Many questions about linear relationships require solution of linear equations or inequalities, such as $50 = 23 + 5.2x$ or $45 - 3.5x < 25$.

**a** What does it mean to solve an equation or inequality?

**b** How could you use tables and graphs of linear functions to solve the following equation and inequality?

  **i.** $50 = 23 + 5.2x$          **ii.** $45 - 3.5x < 25$

**c** How can you check a solution to an equation or inequality?

*Be prepared to share your ideas with the class.*

Jose Luis Pelaez/CORBIS

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# ✔Check Your Understanding

Bronco Electronics is a regional distributor of electronic products specializing in graphing calculators. When an order is received, the shipping department packs the calculators in a box. The shipping cost $C$ is a function of the number $n$ of calculators in the box. It can be calculated using the function $C = 4.95 + 1.25n$.

Use your graphing calculator or computer software to make a table and a graph showing the relation between the number of calculators in a box and shipping costs for that box. Include information for 0 to 20 calculators. Use the table, graph, or cost function rule to answer the following questions.

**a.** How much would it cost to ship an empty box? How is that information shown in the table, the graph, and the cost function rule?

**b.** How much does the addition of a single calculator add to the cost of shipping a box? How is that information shown in the table, the graph, and the cost function rule?

**c.** Write and solve equations or inequalities to answer the following questions about Bronco Electronics shipping costs.

   **i.** What is the cost of shipping 8 calculators?

   **ii.** If the shipping cost is $17.45, how many calculators are in the box?

   **iii.** How many calculators can be shipped if the cost is to be held to at most $25?

**d.** What questions about shipping costs could be answered by solving:

   **i.** $27.45 = 4.95 + 1.25n$?

   **ii.** $4.95 + 1.25n \leq 10$?

## Investigation 2   Using Your Head

It is often possible to solve problems that involve linear equations without the use of tables, graphs, or computer algebra systems. Solving equations by symbolic reasoning is called solving *algebraically*. For example, to solve $3x + 12 = 45$ algebraically you might reason like one of these students.

**Michael**

The equation tells me to multiply $x$ by 3, then add 12, to get 45. To find out what value of $x$ gives me 45, I have to undo those operations.

That means, starting with 45, I can subtract 12 and then divide by 3 to get

$$x = \frac{(45 - 12)}{3}$$

$$= \frac{33}{3}$$

$$= 11$$

**Natasha**

I need a value of $x$ making $3x + 12 = 45$, so the left and right sides are balanced.

If I subtract 12 from both sides, the sides will remain balanced. So, $3x = 33$.

If I divide both sides by 3, the sides will remain balanced. So, $x = 11$.

Tom Pantages

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As you work on the problems in this investigation, think about these questions:

*Why does solving linear equations by reasoning like that of Natasha and Michael make sense?*

*How can reasoning like that of Natasha and Michael be used to solve other linear equations algebraically?*

**1** Analyze the reasoning strategies used by Natasha and Michael by answering the following questions.

   **a.** Why did Natasha subtract 12 from both sides? Why didn't she add 12 to both sides? What if she subtracted 10 from both sides?

   **b.** Why did Natasha divide both sides by 3?

   **c.** What did Michael mean by "undoing" the operations?

   **d.** Why did Michael subtract 12 and then divide by 3? Why not divide by 3 and then subtract 12?

   **e.** Both students found that $x = 11$. How can you be sure the answer is correct?

**2** Solve the equation $8x + 20 = 116$ algebraically in a way that makes sense to you. Check your answer.

**3** A calculator can help with the arithmetic involved in solving equations.

   **a.** When one student used her calculator to solve an equation by undoing the operations, her screen looked like that at the left. What equation could she have been solving?

   **b.** What would appear on your screen if you used a calculator to solve the equation $30x + 50 = 120$ by the "undoing" method?

   **c.** What would appear on your screen if you solved $30x + 50 = 120$ in just one step?

```
35-15
           20
Ans/5
            4
```

**4** Profit $P$ (in dollars) at Skate World is given by $P = 5n - 2{,}000$, where $n$ is the number of customers in a week. Solve each of the following equations algebraically and be prepared to explain your reasoning. Explain what the result tells about the number of customers and Skate World's profit.

   **a.** $-500 = 5n - 2{,}000$

   **b.** $0 = 5n - 2{,}000$

   **c.** $1{,}250 = 5n - 2{,}000$

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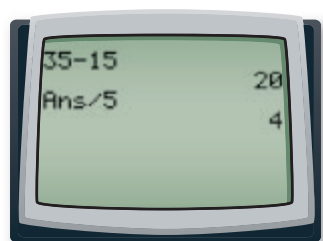
**5** Martin and Anne experimented with the strength of different springs. They found that the length of one spring was a function of the weight upon it according to the function $L = 9.8 - 1.2w$. The length was measured in inches and the weight in pounds. To determine the weight needed to compress the spring to a length of 5 inches, they reasoned as follows.

We need to solve $9.8 - 1.2w = 5$.

**Martin:**
If $9.8 - 1.2w = 5$, then
$9.8 = 5 + 1.2w$.
Then $4.8 = 1.2w$.
So, $w = \frac{4.8}{12}$,
or $w = 4$.

**Anne:**
If $9.8 - 1.2w = 5$, then
$9.8 - 5 = 1.2w$.
This means that $4.8 = 1.2w$.
So, $w = \frac{4.8}{12}$,
or $w = 4$.

**a.** Is each step of their reasoning correct? If so, how would you justify each step? If not, which step(s) contains errors and what are those errors?

**b.** What does the answer tell about the spring?

**6** Bronco Electronics received bids from two shipping companies. For shipping $n$ calculators, Speedy Package Express would charge $3 + 2.25n$ dollars. The Fly-By-Night Express would charge $4 + 2n$ dollars. Solve the equation $3 + 2.25n = 4 + 2n$. Explain what the solution tells about the shipping bids.

## Summarize
### the Mathematics

It is often relatively easy to solve problems involving linear equations algebraically.

**a** Suppose you are going to tell someone how to solve an equation like $43 = 7 - 4x$ algebraically. What steps would you recommend?

**b** When would you recommend solving an equation algebraically? When would you advise use of a table, graph, or computer algebra system?

*Be prepared to explain your procedure and thinking with the class.*

# ✔ Check Your Understanding

When a soccer ball, volleyball, or tennis ball is hit into the air, its upward velocity changes as time passes. **Velocity** is a measure of both speed and direction. The ball slows down as it reaches its maximum height and then speeds up in its return flight toward the ground. On its way up, the ball has a positive velocity, and on its way down it has a negative velocity. Suppose the upward velocity of a high volleyball serve is given by the function:

$$v = 64 - 32t$$

where $t$ is time in seconds and $v$ is velocity in feet per second.

**a.** Solve each of the following equations algebraically. Show your reasoning and explain what each solution tells about the flight of the ball.

   **i.** $16 = 64 - 32t$

   **ii.** $64 - 32t = -24$

   **iii.** $64 - 32t = 0$

   **iv.** $96 = 64 - 32t$

**b.** If you were to estimate solutions for the equations in Part a using a table of $(t, v)$ values for the function $v = 64 - 32t$, what table entries would provide each solution? Record your answers in a table like this.

| Equation | $t$ | $v$ |
|----------|-----|-----|
| i | | |
| ii | | |
| iii | | |
| iv | | |

**c.** If you were to estimate solutions for the equations in Part a using a graph of $v = 64 - 32t$, what points would provide each solution? Record your answers on a sketch of the graph, labeling each point with its equation number and coordinates.

**d.** What is the rate of change in velocity as time passes? What units describe this rate of change? (This rate of change represents the *acceleration* due to gravity.)

## Investigation ③    Using Your Head ... More or Less

The reasoning you used in Investigation 2 to solve linear equations can be applied to solve linear inequalities algebraically. However, unlike equations, the direction of an inequality matters. If $x = 2$, then $2 = x$. On the other hand, $3 < 5$ is true but $5 < 3$ is not. As you work through the problems of this investigation, make notes of answers to the following question:

*How can you solve a linear inequality algebraically?*

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**①** Begin by exploring the effect of multiplying both sides of an equality by a negative number.

   **a.** Consider the following true statements.

$$3 < 7 \qquad -2 < 1 \qquad -8 < -4$$

   For each statement, multiply the number on each side by $-1$. Then indicate the relationship between the resulting numbers using $<$ or $>$.

   **b.** Based on your observations from Part a, complete the statement below.

$$\text{If } a < b, \text{ then } (-1)a \underline{\quad ? \quad} (-1)b.$$

   **c.** Next, consider relations of the form $c > d$ and multiplication by $-1$. Test several examples and then make a conjecture to complete the statement below.

$$\text{If } c > d, \text{ then } (-1)c \underline{\quad ? \quad} (-1)d.$$

**②** Pairs of numbers are listed below. For each pair, describe how it can be obtained *from the pair above it*. Then indicate whether the direction of the inequality stays the same or reverses. The first two examples have been done for you.

| | **Inequality operation** | **Inequality direction** |
|---|---|---|
| **9 > 4** | | |
| 12 _>_ 7 | add 3 to both sides | stays the same |
| 24 _>_ 14 | multiply both sides by 2 | stays the same |
| **a.** 20 _?_ 10 | | |
| **b.** −4 _?_ −2 | | |
| **c.** −2 _?_ −1 | | |
| **d.** 8 _?_ 4 | | |
| **e.** 6 _?_ 2 | | |
| **f.** −18 _?_ −6 | | |
| **g.** 3 _?_ 1 | | |
| **h.** 21 _?_ 7 | | |

**③** Look back at your answers to Problem 2 and identify cases where operations reversed the direction of inequality.

   **a.** What operations seem to cause this reversal of inequality relationships?

   **b.** See if you can explain why it makes sense for those operations to reverse inequality relationships. Compare your ideas with those of your classmates and resolve any differences.

**④** In Investigation 1, you saw that the percentages of male and female doctors can be estimated from the number of years since 1960 using the following functions.

   **Percentage of Male Doctors:**

$$y_1 = 98 - 0.54t$$

   **Percentage of Female Doctors:**

$$y_2 = 2 + 0.54t$$

When their class was asked, "For how long will the majority of U.S. medical doctors be male?",

Taylor wrote this inequality: $98 - 0.54t > 50$.
Jamie wrote this inequality: $2 + 0.54t < 50$.

a. Explain the reasoning that Jamie may have used to create her inequality. Do you think that the solution to either inequality will answer the question? Why or why not?

b. Taylor's and Jamie's solutions are given below. Based on what you know about the percentages of male and female doctors in the United States, which answer makes more sense? Why?

| Taylor's solution: | Jamie's solution: |
|---|---|
| I need to solve $98 - 0.5t > 50$. | I need to solve $2 + 0.54t < 50$. |
| Subtract 98 from both sides: $-0.54t > -48$ | Subtract 2 from both sides: $0.54t < 48$ |
| Then divide both sides by $-0.54$: $t > 88.9$ | Then divide both sides by $0.54$: $t < 88.9$ |
| So the majority of U.S. medical doctors will be males beginning approximately 89 years from 1960, or after 2049. | So the majority of U.S. medical doctors will be males for approximately 89 years from 1960, or until 2049. |

c. What is the error in the incorrect solution?

5. Solve the following linear inequalities using reasoning similar to that used in solving simple linear equations algebraically. Pay careful attention to the direction of the inequality. Be sure to check your solutions.

a. $1.5t - 150 > 450$    b. $4.95 + 1.25n \le 10$    c. $45 - 3.5x < 25$

d. $32 \le 6p - 10$    e. $100 > 250 - 7.5d$

## Summarize the Mathematics

Just as with linear equations, it is often relatively easy to solve linear inequalities algebraically.

a. Suppose you are going to tell someone how to solve an inequality like $7 - 4x > 43$ algebraically. What steps would you recommend? Why?

b. How would you check your solution to an inequality like $7 - 4x > 43$? To an inequality like $7 - 4x \ge 43$?

c. How is solving a linear inequality algebraically similar to, and different from, solving a linear equation algebraically?

d. When would you recommend solving an inequality like the ones you've seen so far algebraically? When would you advise use of a table, graph, or computer algebra system?

*Be prepared to explain your procedures and reasoning.*

Michael Keller/CORBIS

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# ✔Check Your Understanding

In Lesson 1, you examined the effects of inflation and depreciation. You developed the following functions to model the change over time in the price of a new video arcade game and the change over time in the resale value of a game purchased new in 2002. Here, $x$ represents years since 2002.

**Price of New Game:**        $y_1 = 500 + 50x$

**Resale Value of Used Game:** $y_2 = 500 - 133x$

Solve each of the following inequalities algebraically.

- Show your reasoning in finding the solutions.
- Check your solutions.
- Explain what each solution tells about game prices.
- Make a table and sketch a graph of the price functions for $0 \le x \le 5$. Highlight the table entries and graph points that indicate the solutions for each inequality.

**a.**  $500 + 50x > 600$

**b.**  $500 - 133x < 100$

**c.**  $700 \ge 500 + 50x$

**d.**  $300 \le 500 - 133x$

## Investigation 4   Making Comparisons

In many problems involving linear functions, the key question asks for comparison of two different functions. For example, in Investigation 1 of this lesson you used linear models to compare the patterns of change in percentage of female and male doctors in the United States. In this investigation, you will examine methods for making sense of situations modeled by other *systems of linear equations*.

Increasing numbers of businesses, including hotels and cafés, are offering access to computers with high-speed Internet. Suppose that while on vacation Jordan would like to read and send e-mail, and two nearby businesses, Surf City Business Center and Byte to Eat Café, advertise their Internet services as shown at the top of the next page.

## Surf City Business Center

Surf the Internet ~
just $3.95 per day
plus $0.05 per minute.

## Byte to Eat Café

Stop in for a byte—
high-speed Internet access
for $2 per day
plus $0.10 per minute.

As you explore the question of which business offers a more economical deal, keep in mind this question:

*How can you represent and solve problems involving comparisons of two linear functions?*

**1** For both businesses, the daily charge is a function of the number of minutes of Internet use.

**a.** For each business, write a rule for calculating the daily charge for any number of minutes.

**b.** What are the daily charges by each business for customers using 30 minutes?

**c.** How many minutes could Jordan spend on the Internet in a day for $10 using the pricing plans for each of the two businesses?

**d.** For what number of minutes of Internet use in a day is Surf City Business Center more economical? For what number of minutes of Internet use in a day is Byte to Eat Café more economical?

**e.** Do you or someone you know use the Internet? For what purposes? Which pricing plan would cost less for this kind of use of the Internet?

**2** To compare the price of Internet access from the two businesses, the key problem is to find the number of minutes for which these two plans give the same daily charge. That means finding a value of $x$ (number of minutes) for which each function gives the same value of $y$ (daily cost).

**a.** Use tables and graphs to find the number of minutes for which the two businesses have the same daily charge. Indicate both the number of minutes and the daily charge.

**b.** When one class discussed their methods for comparing the price of Internet access from the two businesses, they concluded, "The key step is to solve the equation $3.95 + 0.05x = 2 + 0.10x$." Is this correct? Explain your reasoning.

**c.** Solve the equation in Part b algebraically. Show your reasoning.

The questions about daily Internet access charges from two different businesses involve comparisons of two linear functions. The functions can be expressed with rules.

**Surf City Business Center:** $y_1 = 3.95 + 0.05x$

**Byte to Eat Café:** $y_2 = 2 + 0.10x$

If you think about what values of $x$ and $y$ will make $y = 3.95 + 0.05x$ and $y = 2 + 0.10x$ true, then you are thinking about $y = 3.95 + 0.05x$ and $y = 2 + 0.10x$ as equations. The pair of linear equations is sometimes called a **system of linear equations**.

**3** Finding the pairs of numbers $x$ and $y$ that satisfy both equations is called **solving the system**.

    **a.** Does the pair (1, 7) satisfy either equation in the system above? If so, what does this solution say about Internet access charges? What about the pairs (10, 3) and (20, 8)?

    **b.** Find a pair of numbers $x$ and $y$ that satisfies both equations. What does this solution say about Internet access charges?

    **c.** Is there another solution of the system, that is, another pair of numbers $(x, y)$ that satisfies both equations? How do you know?

Thinking about comparing costs is helpful when developing strategies for solving systems of linear equations. You can use the same strategies for solving when you do not know what $x$ and $y$ represent.

**4** Use tables, graphs, or algebraic reasoning to solve each system. Use each method of solution at least once. Check each solution by substituting the values of $x$ and $y$ into both original equations. If a system does not have a solution, explain why.

    **a.** $\begin{cases} y = 2x + 5 \\ y = 3x + 1 \end{cases}$

    **b.** $\begin{cases} y = 10 - 1.6x \\ y = 2 + 0.4x \end{cases}$

    **c.** $\begin{cases} y = 1.5x + 2 \\ y = 5 + 1.5x \end{cases}$

    **d.** $\begin{cases} y = 2(3 + 0.8x) \\ y = 1.6x + 6 \end{cases}$

**5** Describe a situation (like the Internet access situation) that involves comparing two linear functions. Set up and solve a system of linear equations that might model the situation and explain what the solution tells you about the situation.

# Summarize
## the Mathematics

In solving a system of linear equations like $y = 5x + 8$ and $y = 14 - 3x$:

**a** What is the goal? How is it similar to and different from the goal in solving the single equation $5x + 8 = 14 - 3x$?

**b** How could the solution to the system be found on a graph of the two functions?

**c** How could the solution be found in a table of $(x, y)$ values for both functions?

**d** How could the solution be found algebraically?

**e** What patterns in the tables, graphs, and equations of a system indicate that there is no $(x, y)$ pair that satisfies both equations? What patterns indicate that there are many $(x, y)$ pairs that satisfy both equations?

*Be prepared to explain your solution methods and reasoning.*

## ✔ Check Your Understanding

Charter-boat fishing for walleyes is popular on Lake Erie. The charges for an eight-hour charter trip are:

| Charter Company | Boat Rental | Charge per Person |
|---|---|---|
| Wally's | $200 | $29 |
| Pike's | $50 | $60 |

Each boat can carry a maximum of ten people in addition to the crew.

**a.** Write rules for calculating the cost for charter service by Wally's and by Pike's.

**b.** Determine which service is more economical for a party of 4 and for a party of 8.

**c.** Assuming you want to minimize your costs, under what circumstances would you choose Wally's charter service? How would you represent your answer symbolically?

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## *Applications*

**1** Parents often weigh their child at regular intervals during the first several months after birth. The data usually can be modeled well with a linear function. For example, the rule $y = 96 + 2.1x$ gives Rachel's weight in ounces as a function of her age in days.

   **a.** How much did Rachel weigh at birth?

   **b.** Make a table and a graph of this function for $0 \leq x \leq 90$ with $\Delta x = 15$.

   **c.** For each equation or inequality below, use the table or graph to estimate the solution of the equation or inequality. Then explain what the solution tells you about Rachel's weight and age.

      **i.** $y = 96 + 2.1(10)$      **ii.** $159 = 96 + 2.1x$

      **iii.** $264 = 96 + 2.1x$      **iv.** $96 + 2.1x \leq 201$

**2** Mary and Jeff both have jobs at their local baseball park selling programs. They get paid $10 per game plus $0.25 for each program they sell.

   **a.** Write a rule for pay earned as a function of number of programs sold.

   **b.** Write equations, inequalities, or calculations that can be used to answer each of the following questions.

      **i.** How many programs does Jeff need to sell to earn $25 per game?

      **ii.** How much will Mary earn if she sells 75 programs at a game?

      **iii.** How many programs does Jeff need to sell to earn at least $35 per game?

   **c.** Produce a table and a graph of the relation between program sales and pay from which the questions in Part b can be answered. Use the graph and the table to estimate the answers.

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**3** Ella works as a server at Pietro's Restaurant. The restaurant owners have a policy of automatically adding a 15% tip on all customers' bills as a courtesy to their servers. Ella works the 4 P.M. to 10 P.M. shift. She is paid $15 per shift plus tips.

  **a.** Write a rule for Ella's evening wage based on the total of her customers' bills. Use your graphing calculator or computer software to produce a table and a graph of this function.

  **b.** If the customers' bills total $310, how much will Ella earn?

  **c.** Write and solve an equation to answer the question, "If Ella's wage last night was $57, what was the total for her customers' bills?"

**4** The Yogurt Shop makes several different flavors of frozen yogurt. Each new batch is 650 ounces, and a typical cone uses 8 ounces. As sales take place, the amount $A$ of each flavor remaining from a fresh batch is a function of the number $n$ of cones of that flavor that have been sold. The function relating amount of yogurt left to number of cones sold is $A = 650 - 8n$.

  **a.** Solve each equation related to sales of chocolate-vanilla swirl yogurt. Show your work. Explain what your solution tells about sales of chocolate-vanilla yogurt and the amount left.

  **i.** $570 = 650 - 8n$

  **ii.** $250 = 650 - 8n$

  **iii.** $A = 650 - 8(42)$

  **b.** Use the function $A = 650 - 8n$ to write and solve equations to answer the following questions.

  **i.** How many cones have been sold if 390 ounces remain?

  **ii.** How much yogurt will be left when 75 cones have been sold?

  **iii.** If the machine shows 370 ounces left, how many cones have been sold?

**5** Victoria can earn as much as $450 as a scorekeeper for a summer basketball league. She learned that she must pay $18 per game for substitutes when she misses a game. So, her summer earnings $E$ will depend on the number of games she misses $g$ according to the function $E = 450 - 18g$. Solve each of the following equations and explain what the solutions tell you about Victoria's summer earnings.

  **a.** $306 = 450 - 18g$

  **b.** $360 = 450 - 18g$

  **c.** $0 = 450 - 18g$

  **d.** $E = 450 - 18(2)$

  **e.** $315 = 450 - 18g$

  **f.** $486 = 450 - 18g$

**6** When people shop for cars or trucks, they usually look closely at data on fuel economy. The data are given as miles per gallon in city and in highway driving. Let $c$ stand for miles per gallon in city driving and $h$ stand for miles per gallon in highway driving. Data from tests of 20 of the most popular American cars and trucks show that the function $h = 1.4 + 1.25c$ is a good model for the relation between the variables. Solve the following equations algebraically. Explain what the results tell you about the relation between city and highway mileage. Be prepared to explain the reasoning you used to find each solution.

   **a.** $35 = 1.4 + 1.25c$

   **b.** $10 = 1.4 + 1.25c$

   **c.** $h = 1.4 + (1.25)(20)$

**7** Describe a problem situation which could be modeled by the function $y = 10 + 4.35x$.

   **a.** What would solving $109 \geq 10 + 4.35x$ mean in your situation?

   **b.** Solve $109 \geq 10 + 4.35x$ algebraically. Then show how the solution could be estimated from a table or graph.

**8** Solve each of the following equations and inequalities algebraically and check your answers. Show the steps in your solutions and in your checks.

   **a.** $25 = 13 + 3x$

   **b.** $74 = 8.5x - 62$

   **c.** $34 + 12x < 76$

   **d.** $76 \geq 34 - 12x$

   **e.** $3{,}141 = 2{,}718 + 42x$

**9** Refer back to the Internet pricing plans for Surf City Business Center and Byte to Eat Café given on page 198. Suppose Surf City Business Center wants to become more competitive for customers looking for high-speed Internet access. The owner decides to change the daily base charge from $3.95 to $2.95, but maintain the $0.05 per minute charge.

   **a.** Write a rule for the daily charges under the new program.

   **b.** How are the graphs of the new and old Internet access charges related?

   **c.** What would be the daily charge for 30 minutes of Internet use using the new program?

   **d.** How many minutes would one need to spend on the Internet in order for Surf City Business Center's new program to be more economical than Byte to Eat Café?

   **e.** If a customer is charged $5.20 for one day's Internet use under the new Surf City Business Center pricing plan, how many minutes did he or she spend online?

   **f.** What would Byte to Eat Café have charged for the same number of minutes?

A.E. Cook

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**10** Surf City Business Center did not notice any large increase in customers when they changed their base daily charge from $3.95 to $2.95. They decided to change it back to $3.95 and reduce the per-minute charge from $0.05 to $0.03.

a. Write a rule that models their new Internet access charge.

b. How are the graphs of this new and the original Internet access charges related?

c. What is the cost of 30 minutes of Internet use under this new plan?

d. For how many minutes of Internet use is this new Surf City Business Center pricing plan competitive with Byte to Eat Café?

e. Compare the cost of Internet access under this plan with that proposed in Applications Task 9. Which plan do you think will attract more customers? Explain your reasoning.

**11** Recall Bronco Electronics, a regional distributor of graphing calculators, from the Check Your Understanding on page 191. Their shipping cost $C$ can be calculated from the number $n$ of calculators in a box using the rule $C = 4.95 + 1.25n$. Bronco Electronics got an offer from a different shipping company. The new company would charge based on the rule $C = 7.45 + 1.00n$. Write and solve equations or inequalities to answer the following questions:

a. For what number of calculators in a box will the two shippers have the same charge?

b. For what number of calculators in a box will the new shipping company's offer be more economical for Bronco Electronics?

**12** From the situations described below, choose two situations that most interest you. Identify the variables involved and write rules describing one of those variables as a function of the other. In each case, determine conditions for which each business is more economical than the other. Show how you compared the costs.

a. A school club decides to have customized T-shirts made. The Clothing Shack will charge $30 for setup costs and $12 for each shirt. The cost of having them made at Clever Creations is a $50 initial fee for the setup and $8 for each T-shirt.

b. Speedy Telegram Service charges a $30 base fee and $0.75 for each letter or symbol. Quick Delivery charges a $25 base fee and $0.90 for each character.

c. Cheezy's Pies charges $5 for a 12-inch sausage pizza and $5 for delivery. The Pizza Palace delivers for free, but they charge $7 for a 12-inch sausage pizza.

d. The *Evening News* has a minimum charge of $4 for up to 3 lines and $1.75 for each additional line of a listing placed in the classified section. The *Morning Journal* charges $8 for the first 5 lines and $1.25 for each additional line.

Tom Pantages

**13** Solve each of the following systems of linear equations and check each solution. Among the four problems, use at least two different solution methods (tables, graphs, or algebraic reasoning).

**a.** $\begin{cases} y = x + 4 \\ y = 2x - 9 \end{cases}$

**b.** $\begin{cases} y = -2x + 18 \\ y = -x + 10 \end{cases}$

**c.** $\begin{cases} y = 3x - 12 \\ y = 1.5x + 3 \end{cases}$

**d.** $\begin{cases} y = x \\ y = -0.4x + 7 \end{cases}$

## *Connections*

**14** Recall the formula for the circumference of a circle:

$$C = \pi d.$$

Write equations or inequalities that can be used to answer the following questions. Then find answers to the questions.

**a.** A 16-inch pizza has a diameter of 16 inches. What is the circumference of a 16-inch pizza?

**b.** The average arm span of a group of 10 first graders is 47 inches. If they hold hands and stretch to form a circle, what will be the approximate diameter of the circle?

**c.** If you have 50 inches of wire, what are the diameters of the circles you could make with all or part of the wire?

**15** There are two especially useful properties of arithmetic operations. The first relates addition and subtraction. The second relates multiplication and division.

- *For any numbers a, b, and c, a + b = c is true if and only if a = c − b.*

- *For any numbers a, b, and c, a × b = c is true if and only if a = c ÷ b and b ≠ 0.*

Erik solved the equation $3x + 12 = 45$ given at the beginning of Investigation 2 as follows:

If 3*x* + 12 = 45, then 3*x* = 45 − 12, or 3*x* = 33.

If 3*x* = 33, then *x* = 33 ÷ 3, or *x* = 11.

**a.** Explain how the above properties of operations can be used to support each step in Erik's reasoning.

**b.** Solve the equation $130 + 30x = 250$ using reasoning like Erik's that relies on the connection between addition and subtraction and the connection between multiplication and division. Check your answer.

**CPMP-Tools**
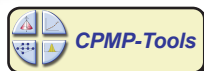
**16** When you know the algebraic operations that you want to use in solving an equation, you can get help with the details from a computer algebra system (CAS). For example, to solve $3x + 11 = 5x + 7$, you can proceed as in the screen below.
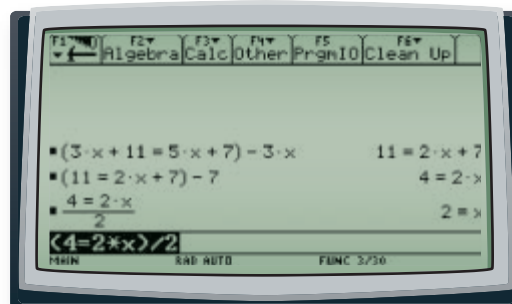


Study this work to figure out what each entered instruction is asking the CAS to do. Then apply your understanding to solve the following equations in a similar way with the CAS available to you. Record the steps you enter at each step in the solution process and the results of those steps.

**a.** $-3x + 72 = 4x - 5$

**b.** $\frac{2}{5}x - \frac{9}{5} = \frac{7}{10}$

**c.** $2.5t - 5.1 = 9.3 - 0.7t$

**17** Here are the first three shapes in a geometric pattern of Xs made from identical squares.



| 1st | 2nd | 3rd |
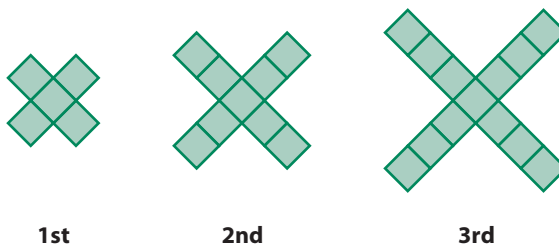
**a.** Write a rule showing how to calculate the number of squares in the *NEXT* shape from the number of squares *NOW*.

**b.** Write a rule for the number $S$ of squares in the $n$th shape.

**c.** Solve these equations and inequalities, and explain what the solution tells you about the pattern.

  **i.** $4n + 1 = 49$     **ii.** $4n + 1 = 81$

  **iii.** $S = 4(8) + 1$     **iv.** $4n + 1 < 100$

**18** The table on page 207 shows winning times for women and men in the Olympic 100-meter freestyle swim for games since 1912.

**a.** Make plots of the (*year*, *winning time*) data for men and for women. Use 0 for the year 1900.

**b.** Find the linear regression model for each data pattern.

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**c.** Which group of athletes has shown a greater improvement in time, men or women? Explain.

**d.** What are the approximate coordinates of the point where the linear regression lines intersect?

**e.** What is the significance of the point of intersection of the two lines in Part d? How much confidence do you have that the lines accurately predict the future? Explain.

### Olympic 100-meter Freestyle Swim Times

| Year | Women's Time (in seconds) | Men's Time (in seconds) | Year | Women's Time (in seconds) | Men's Time (in seconds) |
|------|---------------------------|-------------------------|------|---------------------------|-------------------------|
| 1912 | 82.2 | 63.4 | 1968 | 60.0 | 52.2 |
| 1920 | 73.6 | 61.4 | 1972 | 58.59 | 51.22 |
| 1924 | 72.4 | 59.0 | 1976 | 55.65 | 49.99 |
| 1928 | 71.0 | 58.6 | 1980 | 54.79 | 50.40 |
| 1932 | 66.8 | 58.2 | 1984 | 55.92 | 49.80 |
| 1936 | 65.9 | 57.6 | 1988 | 54.93 | 48.63 |
| 1948 | 66.3 | 57.3 | 1992 | 54.64 | 49.02 |
| 1952 | 66.8 | 57.4 | 1996 | 54.50 | 48.74 |
| 1956 | 62.0 | 55.4 | 2000 | 53.83 | 48.30 |
| 1960 | 61.2 | 55.2 | 2004 | 53.84 | 48.17 |
| 1964 | 59.5 | 53.4 | | | |

**Source:** *The World Almanac and Book of Facts 2003.* Mahwah, NJ: World Almanac Education Group, Inc. 2003; www.olympics.com

## *Reflections*

**19** The function $y = 43 + 5x$ and the equation $78 = 43 + 5x$ are closely related to each other.

**a.** How can you use $y = 43 + 5x$ to solve $78 = 43 + 5x$?

**b.** What does solving $78 = 43 + 5x$ tell you about $y = 43 + 5x$?

**20** When solving equations or inequalities that model real situations, why should you check not only the solution, but also whether the solution makes sense? Illustrate your thinking with an example.

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**21** Consider Mary and Jeff's pay possibilities for selling programs at the ballpark. The rule $P = 10 + 0.25s$ gives their pay $P$ in dollars for selling $s$ programs in a night.

    **a.** What operations are needed to calculate the pay for selling 36 programs in a single night?

    **b.** What operations are needed to solve the equation $10 + 0.25s = 19$? What will the solution tell you?

    **c.** In what sense do the operations in Part b "undo" the operations in Part a?

    **d.** How does the order in which you do the operations in Part a compare with the order in Part b? Why does this make sense?

    **e.** How, if at all, does the procedure for solving change if you are asked to solve the inequality $10 + 0.25s \geq 19$? How does the meaning of the solution change?

**22** How could you use a graph of the function $y = 7 - 4x$ to decide, without calculation or algebraic solution, whether the solutions to $7 - 4x > 43$ will be an inequality like $x < a$ or like $x > a$ and whether $a$ will be positive or negative?

**23** Consider the inequality $2x + 8 > 5x - 4$.

    **a.** What are two different, but reasonable, first steps in solving $2x + 8 > 5x - 4$?

    **b.** What does the solution to the inequality tell you about the graphs of $y = 2x + 8$ and $y = 5x - 4$?

    **c.** What does the solution tell you about tables of values for $y = 2x + 8$ and $y = 5x - 4$?

**24** When asked to solve the system of linear equations

$$\begin{cases} y = 2x + 9 \\ y = 5x - 18 \end{cases}$$

Sabrina reasoned as follows:

> I want $x$ so that $2x + 9 = 5x - 18$.
>
> Adding 18 to each side of that equation gives me $2x + 27 = 5x$, and the sides remain balanced.
>
> Subtracting $2x$ from each side of the new equation gives $27 = 3x$, and the sides remain balanced.
>
> Dividing each side of that equation by 3 gives $x = 9$, and the sides remain balanced.
>
> If $x = 9$, then one equation is $y = 2(9) + 9$ and the other equation is $y = 5(9) - 18$. Both equations give $y = 27$.
>
> The solution of the system must be $x = 9$ and $y = 27$.

    **a.** Do you agree with each step of her reasoning? Why or why not?

    **b.** Use reasoning like Sabrina's to solve the following system of linear equations.

$$\begin{cases} y = 8x + 3 \\ y = 2x - 9 \end{cases}$$

## *Extensions*

**25** Refer back to Applications Task 3. Suppose a new policy at the restaurant applies an automatic service charge of 15% to each bill. Servers will receive $20 per shift plus 10% of the bills for customers they serve. Busers will receive $25 per shift plus 5% of the bills for customers at tables they serve.

CPMP-Tools

  **a.** Write rules for the functions that show how:

   **i.** a server's daily earnings depend on the total of bills for customers at tables he or she serves.

   **ii.** busers' daily earnings depend on the total of bills for customers at tables he or she serves.

   Graph these two functions on the same coordinate axes.

  **b.** Write three questions about the wages for wait staff and busers. Write equations or inequalities corresponding to your questions. Then solve the equations or inequalities and answer the questions. Show how you arrived at your solution for each equation or inequality.

**26** The diagram at the right shows graphs of two functions:

$y = x + 3$ and $y = x^2 - 3$.

Reproduce the graphs on your graphing calculator or computer and use the graphs to solve each equation or inequality.



  **a.** $x + 3 = x^2 - 3$

  **b.** $x + 3 \geq x^2 - 3$

  **c.** $x + 3 < x^2 - 3$
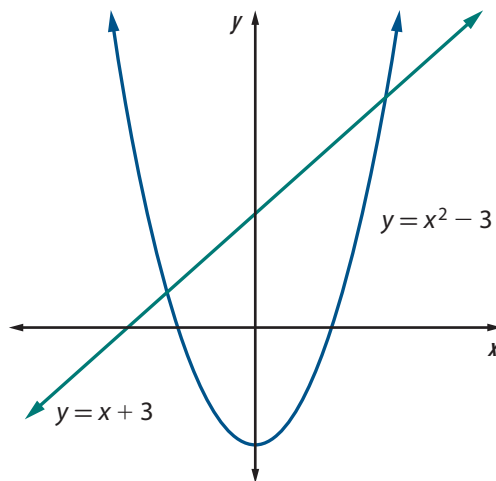
**27** One linear function relating grams of fat $F$ and calories $C$ in popular "lite" menu items of fast-food restaurants is given by $C = 300 + 16(F - 10)$. Solve each equation or inequality below, and explain what your solution tells you about grams of fat and calories in fast-food. Use each of the following strategies at least once.

  • Use a graph of the function.

  • Use a table for the function.

  • Use algebraic reasoning, as in the examples of this lesson.

  **a.** $430 = 300 + 16(F - 10)$

  **b.** $685 = 300 + 16(F - 10)$

  **c.** $140 = 300 + 16(F - 10)$

  **d.** $685 \geq 300 + 16(F - 10)$

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**28** Any linear function can be described by a rule of the form $y = a + bx$. Explain, with sketches, how to solve each of the following using graphs of linear functions. Describe the possible number of solutions. Assume $b \neq 0$ and $d \neq 0$.

**a.** equations of the form $c = a + bx$

**b.** inequalities of the form $c \leq a + bx$

**c.** equations of the form $a + bx = c + dx$

**d.** inequalities of the form $a + bx \leq c + dx$

**29** The student government association at the Baltimore Freedom Academy wanted to order Fall Festival T-shirts for all students. Thrifty Designs charges a one-time art fee of $20 and then $6.25 per shirt.

**a.** What rule shows how to calculate the cost $c_1$ of purchasing $n$ shirts from Thrifty Designs?

**b.** For each of the following questions:

- Write an equation or inequality with solutions that will answer the question.
- Explain how the solution for the equation or inequality is shown in a table or graph of $(n, c_1)$ values.
- Write an answer to the given question about T-shirt purchase.

 **i.** How much would it cost to buy T-shirts for 250 people?

 **ii.** How many T-shirts could be purchased for $1,000?

**c.** Suppose that Tees and More quotes a cost of $1,540 for 250 T-shirts and $1,000 for 160 T-shirts.

 **i.** If the cost of T-shirts from Tees and More is a linear function of the number of shirts purchased, what rule shows the relationship between number of shirts $n$ and cost $c_2$?

 **ii.** What one-time art fee and cost per shirt are implied by the Tees and More price quotation?

 **iii.** Write and solve an inequality that answers the question, "For what numbers of T-shirts will Tees and More be less expensive than Thrifty Designs?" Be sure to explain how the solution is shown in a table or graph of $(n, c_2)$ values.

**30** Refer back to the Internet access pricing plans for Surf City Business Center and Byte to Eat Café given on page 198. Suppose Surf City Business Center decides to lower its base daily charge to $2.95 but is unsure what to charge per call. They want to advertise daily charges that are lower than Byte to Eat Café if one spends more than 20 minutes online per day.

**a.** To meet their goal, at what point will the Surf City Business Center graph need to cross the Byte to Eat Café graph?

**b.** What charge per minute by Surf City Business Center will meet that condition?

**c.** Suppose a customer spends 60 minutes online per day. By how much is the new Surf City Business Center plan lower than the Byte to Eat Café plan for this many minutes?

**d.** If a person spends only 10 minutes online per day, how much less will he or she spend by using Byte to Eat Café rather than the new Surf City Business Center plan?

**31** Refer back to the Check Your Understanding on page 200. Suppose it was noticed that most fishing parties coming to the dock were 4 or fewer persons.

**a.** How should Wally revise his boat rental fee so that his rates are lower than the competition's (Pike's) for parties of 3 or more? Write a rule for the new rate system.

**b.** How much less would a party of four pay by hiring Wally's charter service instead of Pike's?

**c.** Which service should you hire for a party of 2? How much will you save?

**d.** Suppose Pike's charter service lowers the per-person rate from $60 to $40. For what size parties would Pike's be less expensive?

**e.** If Wally wants to change his per-person rate so that both services charge the same for parties of 4, what per-person rate should Wally charge? Write a rule that models the new rate structure.

**32** Create a linear system relating cost to number of uses of a service for which Company A's rate per service is 1.5 times that of Company B's, but Company B's service is not more economical until 15 services have been performed.

## *Review*

**33** Match each triangle description with the sketch(es) to which it applies.

**a.** An acute triangle

**b.** A scalene triangle

**c.** An obtuse triangle

**d.** An isosceles triangle

**e.** An equilateral triangle

**f.** A right scalene triangle

**g.** An obtuse isosceles triangle
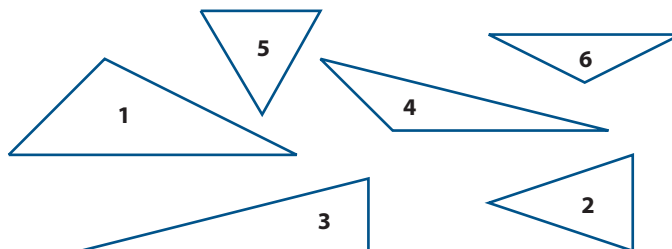
**34** Use the fact that 5% of $50,000 = $2,500 to calculate the following percentages.

**a.** 0.5% of $50,000

**b.** 5.5% of $50,000

**c.** 105% of $50,000

**d.** 95% of $50,000

**35** Use the fact that $13 \times 14 = 182$ to mentally calculate the value for each of the following expressions.

    **a.** $(-13)(-14)$                    **b.** $(14)(-13)$

    **c.** $(13)(14) + (-13)(14)$      **d.** $\dfrac{(13)(-14)}{(-14)(-13)}$

**36** Write an equation for the line passing through the points with coordinates $(-1, -2)$ and $(2, 0)$.

**37** Find the area and perimeter of each figure. Assume that all angles that look like right angles are right angles and all segments that look parallel are parallel.

    **a.**

12 cm

8 cm

    **b.**

8 in.

6 in.

    **c.**

15 m

10 m

6 m

    **d.**

8 mm

10 mm

6 mm

12 mm
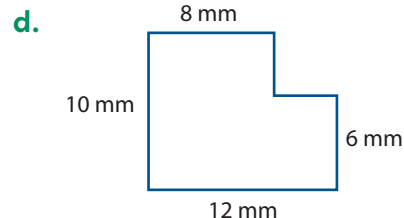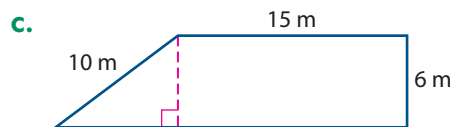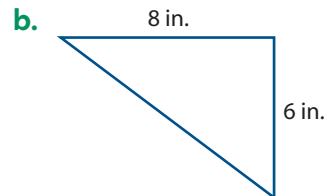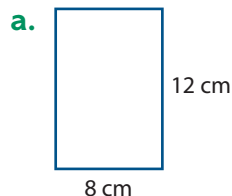
**38** List all of the 2-digit numbers that can be made from the digits 3, 4, and 5. Digits may be repeated.

    **a.** Suppose that you randomly choose one of the numbers you listed. What is the probability that it is divisible by 5?

    **b.** Suppose that you randomly choose one of the numbers you listed. What is the probability that it is divisible by 3?

    **c.** Suppose that you randomly choose one of the numbers you listed. What is the probability that it is divisible by 5 or 3?

    **d.** Suppose that you randomly choose one of the numbers you listed. What is the probability that it is divisible by 5 and 3?

**39** The following four expressions look very similar.

$$2 - x - 5 \qquad 2 - (x - 5) \qquad 2 - (x + 5) \qquad 2 - 5 - x$$

    **a.** Substitute $x = 0$ into each expression to find the value for each expression.

    **b.** Substitute $x = -1$ into each expression to find the value for each expression.

    **c.** Substitute $x = 1$ into each expression to find the value for each expression.

    **d.** Substitute $x = 2$ into each expression to find the value for each expression.

    **e.** Which of the above expressions will always have the same value when you substitute the same number for $x$ in the expressions? Explain.

**40** Pentagon *ABCDE* is congruent to pentagon *PQRST*. Find each indicated angle measure or side length.
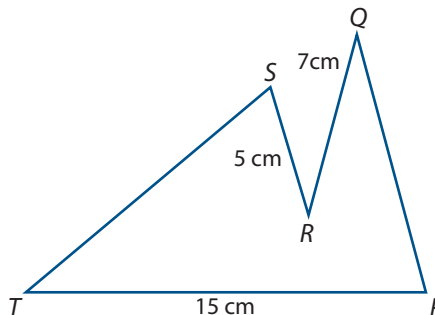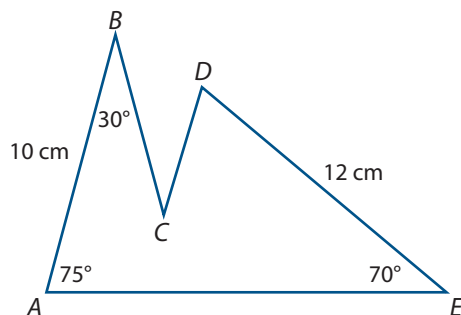


a. *BC*

b. m∠*T*

c. *AE*

d. m∠*P*

e. *ST*

f. *QP*

**41** Tom and Jenny each drew a line to fit the same linear pattern of data. They wrote rules to make predictions for the value of *y* for different values of *x*. Complete the table below to compare their predictions.

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| (Tom's Rule) $y = 0.5x + 7$ | 7 | 7.5 | | |
| (Jenny's Rule) $y = 0.6x + 7$ | 7 | 7.6 | | |
| Difference in Predictions | 0 | 0.1 | | |

Why does the difference in predictions change? When is the difference greater than 0.5?

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# *Equivalent Expressions*

Entertainment is a big business in the United States—from network television, movies, and concert tours to local school plays and musical shows. Each live or recorded performance is prepared with weeks, months, or even years of creative work and business planning.

For example, a recording label might have the following costs to produce a CD by a popular music artist.

- $100,000 to record the tracks;
- $1.50 per copy for materials and reproduction;
- $2.25 per copy for royalties to the writers, producers, and performers.

The record label might receive income of about $5 per copy from the stores that sell the CD.

The record label's profit on a CD is a function of the number of copies that are made and sold. A function rule for profit gives an *expression* for calculating profit and assigns a variable for the output. This lesson will focus on expressions for calculating various quantities.

**a** Using the given numbers, how would you calculate the label's net profit for 100,000 copies made and sold? For 1 million copies made and sold? For $n$ copies made and sold?

**b** One group of students wrote the expression $5n - (3.75n + 100,000)$ to calculate the label's net profit on sales of $n$ copies. Is this correct? How do you know? Why might they have expressed profit this way?

**c** Another group of students wrote the expression $-100,000 + 1.25n$ to calculate the label's net profit on sales of $n$ copies. Is this correct? How do you know? Why might they have expressed profit this way?

**d** Could you represent the profit for $n$ copies in other different ways?

**e** How could you convince another student that two different expressions represent the same quantity? Consider the expressions for profit you produced or those given in Parts b and c.

Two expressions are **equivalent** if they produce identical outputs when given identical inputs. In this lesson, you will develop your ability to recognize, write, and reason with equivalent linear expressions.

## Investigation 1  Different, Yet the Same

Your thinking about possible profit rules for a new CD release showed an important fact about linear expressions: Several different expressions can each represent the same quantity.

Tables and graphs are one way to explore whether two expressions are equivalent, but it is helpful to be able to tell by looking when two expressions are equivalent. As you work on the problems of this investigation, keep in mind the following question:

*What operations on linear expressions lead to different, but equivalent, expressions?*

**Movie Production** Studios that make motion pictures deal with many of the same cost and income questions as music producers. Contracts sometimes designate parts of the income from a movie to the writers, directors, and actors. Suppose that for one film those payments are:

4% to the writer of the screenplay;
6% to the director;
15% to the leading actors.

**1** What payments will go to the writer, the director, the leading actors, and to all these people combined in the following situations?

  **a.** The studio receives income of $25 million from the film.

  **b.** The studio receives income of $50 million from the film.

**2** Suppose the studio receives income of $I$ million dollars from the film.

  **a.** Write an expression for the total payment to the writer, the director, and the leading actors in a form that shows the breakdown to each person or group.

  **b.** Write another expression for the total payment that shows the combined percent of the film income that is paid out to the writer, the director, and the leading actors.

**3** A movie studio will have other costs too. For example, there will be costs for shooting and editing the film. Suppose those costs are $20 million.

  **a.** Assume that the $20 million for shooting and editing the film and the payments to the writer, the director, and the leading actors are the only costs for the film. What will the studio's profit be if the income from the film is $50 million?

  **b.** Consider the studio's profit (in millions of dollars) when the income from the film is $I$ million dollars.

  **i.** Write an expression for calculating the studio's profit that shows the separate payments to the writer, the director, and the leading actors.

  **ii.** Write another expression for calculating the studio's profit that combines the payments to the writer, the director, and the leading actors.

  **iii.** Is the following expression for calculating the studio's profit correct? How do you know?

$$I - (20 + 0.25I)$$

  **iv.** Write another expression for calculating the studio's profit and explain what that form shows.

**Movie Theaters** For theaters, there are two main sources of income. Money is collected from ticket sales and from concession stand sales.

Suppose that a theater charges $8 for each admission ticket, and concession stand income averages $3 per person.

**(4) Income** Consider the theater income during a month when they have $n$ customers.

**a.** Write an expression for calculating the theater's income that shows separately the income from ticket sales and the income from concession stand sales.

**b.** Write another expression for calculating income that shows the total income received per person.

**(5) Expenses** Suppose that the theater has to send 35% of its income from ticket sales to the movie studio releasing the film. The theater's costs for maintaining the concession stand stock average about 15% of concession stand sales. Suppose also that the theater has to pay rent, electricity, and staff salaries of about $15,000 per month.

**a.** Consider the theater's expenses when the theater has $n$ customers during a month.

  **i.** How much will the theater have to send to movie studios?

  **ii.** How much will the theater have to spend to restock the concession stand?

  **iii.** How much will the theater have to spend for rent, electricity, and staff salaries?

**b.** Write two expressions for calculating the theater's total expenses, one that shows the breakdown of expenses and another that is as short as possible.

**(6) Profit** Consider next the theater's profit for a month in which the theater has $n$ customers.

**a.** Write an expression for calculating the theater's profit that shows each component of the income and each component of the expenses.

**b.** Write another expression for calculating the theater's profit that shows the total income minus the total expenses.

**c.** Write another expression for calculating the theater's profit that is as short as possible.

**(7) Taxes** The movie theater charges $8 per admission ticket sold and receives an average of $3 per person from the concession stand. The theater has to pay taxes on its receipts. Suppose the theater has to pay taxes equal to 6% of its receipts.

**a.** Consider the tax due if the theater has 1,000 customers.

  **i.** Calculate the tax due for ticket sales and the tax due for concession stand sales, then calculate the total tax due.

  **ii.** Calculate the total receipts from ticket sales and concession stand sales combined, then calculate the tax due.

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**b.** Write two expressions for calculating the tax due if the theater has $n$ customers, one for each way of calculating the tax due described in Part a.

**8** In Problem 6, you wrote expressions for the monthly theater profit after all operating expenses. A new proposal will tax profits only, but at 8%. Here is one expression for the tax due under this new proposal.

$$0.08(7.75n - 15,000)$$

**a.** Is the expression correct? How can you be sure?

**b.** Write an expression for calculating the tax due under the new proposal that is as short as possible. Show how you obtained your expression. Explain how you could check that your expression is equivalent to the one given above.

## Summarize the Mathematics

In many situations, two people can suggest expressions for linear functions that look quite different but are equivalent. For example, these two symbolic expressions for linear functions are equivalent.

$$15x - (12 + 7x) \quad \text{and} \quad 8x - 12$$

**a** What does it mean for these two expressions to be equivalent?

**b** How could you test the equivalence of these two expressions using tables and graphs?

**c** Explain how you might reason from the first expression to produce the second expression.

*Be prepared to explain your responses to the entire class.*

## ✔️ Check Your Understanding

Many college basketball teams play in winter tournaments sponsored by businesses that want the advertising opportunity. For one such tournament, the projected income and expenses are as follows.

- Income is $60 per ticket sold, $75,000 from television and radio broadcast rights, and $5 per person from concession stand sales.
- Expenses are $200,000 for the colleges, $50,000 for rent of the arena and its staff, and a tax of $2.50 per ticket sold.

**a.** Find the projected income, expenses, and profit if 15,000 tickets are sold for the tournament.

**b.** Write two equivalent expressions for tournament income if $n$ tickets are sold. In one expression, show each source of income. In the other, rearrange and combine the income components to give the shortest possible expression.

**c.** Write two equivalent expressions for tournament expenses if $n$ tickets are sold. In one expression, show each source of expense. In the other, rearrange and combine the expense components to give the shortest possible expression.

**d.** Write two equivalent expressions for tournament profit if $n$ tickets are sold. In one expression, show income separate from expenses. In the other, rearrange and combine components to give the shortest possible expression.

## Investigation 2 — The Same, Yet Different

In Investigation 1, you translated information about variables into expressions and then into different, but equivalent, expressions. You used facts about the numbers and variables involved to guide and check the writing of new equivalent symbolic expressions.

The examples in Investigation 1 suggest some ways to rewrite symbolic expressions that will produce equivalent forms, regardless of the situation being modeled. Think about how you can rewrite expressions involving variables in equivalent forms even if you do not know what the variables represent. As you work on the problems of this investigation, look for answers to this question:

*How can algebraic properties of numbers and operations be used to verify the equivalence of expressions and to write equivalent expressions?*

**1** These expressions might represent the profit for a given number of sales. Using your thinking from Investigation 1 as a guide, write at least two different but equivalent expressions for each.

**a.** $8x - 3x - 2x - 50$      **b.** $6a - (20 + 4a)$

**c.** $0.8(10n - 30)$      **d.** $t - 20 - 0.3t$

**2** Think about how you might convince someone else that the expressions you wrote in Problem 1 are, in fact, equivalent.

**a.** How might you use tables and graphs to support your claim?

**b.** How might you argue that two expressions are equivalent without the use of tables or graphs? What kind of evidence do you find more convincing? Why?

**3** Determine which of the following pairs of expressions are equivalent. If a pair of expressions is equivalent, explain how you might justify the equivalence. If a pair is not equivalent, show that the pair is not equivalent.

**a.** $3.2x + 5.4x$ and $8.6x$      **b.** $3(x - 2)$ and $6 - 3x$

**c.** $4y + 7y - 12$ and $-12 + 11y$     **d.** $7x + 14$ and $7(x + 2)$

**e.** $8x - 2(x - 3)$ and $6x - 6$      **f.** $3x + 7y - 21$ and $10xy + (-21)$

**g.** $\dfrac{8y + 12}{4}$ and $2y + 3$      **h.** $x + 4$ and $3(2x + 1) - 5x + 1$

There are five properties of numbers and operations that are especially helpful in transforming algebraic expressions to useful equivalent forms.

- **Distributive Property of Multiplication over Addition**—For any numbers $a$, $b$, and $c$:

$$a(b + c) = ab + ac \text{ and } ac + bc = (a + b)c.$$

This property can be applied to write expressions with or without the use of parentheses. For example,

$$5(2x + 3) = 10x + 15.$$

The distributive property can also be applied to the sums of products with common factors. For example,

$$3x + 7x = (3 + 7)x \qquad \text{and} \qquad 6\pi + 8\pi = (6 + 8)\pi$$
$$= 10x \qquad\qquad\qquad\qquad\qquad = 14\pi.$$

- **Commutative Property of Addition**—For any numbers $a$ and $b$:

$$a + b = b + a.$$

- **Associative Property of Addition**—For any numbers $a$, $b$, and $c$:

$$a + (b + c) = (a + b) + c.$$

The commutative and associative properties are often used together to rearrange the addends in an expression. For example,

$$3x + 5 + 4x + 7 = (3x + 4x) + (5 + 7)$$
$$= 7x + 12.$$

- **Connecting Addition and Subtraction**—For any numbers $a$ and $b$:

$$a - b = a + (-b).$$

This property can be applied to rewrite an expression that involves subtraction so that the terms involved can be rearranged using the commutative and associative properties of addition. For example,

$$4x - 3 - 5x + 7 = 4x + (-3) + (-5x) + 7$$
$$= 4x + (-5x) + (-3) + 7$$
$$= -x + 4.$$

The property can also be used with the distributive property to expand a product that involves subtraction. For example,

$$5(2x - 3) = 5(2x + (-3))$$
$$= 10x + (-15)$$
$$= 10x - 15.$$

- **Connecting Multiplication and Division**—For any numbers $a$ and $b$ with $b \neq 0$:

$$\frac{a}{b} = a \cdot \frac{1}{b}.$$

This property can be combined with the distributive property to rewrite an expression that involves division. For example,

$$\frac{20 - 15x}{5} = (20 - 15x)\frac{1}{5}$$
$$= 4 - 3x.$$

**4** When you are given an expression and asked to write an equivalent expression that does not contain parentheses, this is called **expanding** the expression. Use the distributive property to rewrite the following expressions in *expanded* form.

a. $4(y + 2)$     b. $(5 - x)(3y)$     c. $-2(y - 3)$

d. $-7(4 + x)$     e. $\dfrac{(16x - 8)}{4}$     f. $\dfrac{1}{3}(2x + 3)$

**5** When you are given an expression and asked to write an equivalent expression that gives a product, this is called **factoring** the expression. For example, in Problem 4 Part a you wrote $4(y + 2) = 4y + 8$. Writing $4y + 8$ as $4(y + 2)$ is said to be writing $4y + 8$ in *factored* form. Use the distributive property to rewrite the following expressions in factored form.

a. $2x + 6$     b. $20 - 5y$     c. $6y - 9$

d. $8 + 12x$     e. $3x + 15y$     f. $xy - 7x$

**6** Using the distributive property to add or subtract products with common factors is called **combining like terms**. Use the distributive property to rewrite the following expressions in equivalent shorter form by combining like terms.

a. $7x + 11x$     b. $7x - 11x$

c. $5 + 3y + 12 + 7y$     d. $2 + 3x - 5 - 7x$

e. $\dfrac{3x}{4} - 2x + \dfrac{x}{4}$     f. $10x - 5y + 3y - 2 - 4x + 6$

**7** Write each of the following expressions in its *simplest* equivalent form by expanding and then combining like terms.

a. $7(3y - 2) + 6y$     b. $5 + 3(x + 4) + 7x$

c. $2 + 3x - 5(1 - 7x)$     d. $10 - (5y + 3)$

e. $10 - \dfrac{15x - 9}{3}$     f. $5(x + 3) - \dfrac{4x + 2}{2}$

g. $7y + 4(3y - 11)$     h. $8(x + 5) - 3(x - 2)$

**8** Write each of the following expressions in equivalent form by combining like terms and then factoring.

a. $7 + 15x + 5 - 6x$     b. $x - 10 + x + 2$

c. $20x + 10 - 5x$     d. $24 - 5x - 4 + 6x - 8 + 2x$

**9** When *simplifying* an expression, it is easy to make mistakes. Some of the pairs of expressions below are equivalent and some are not. If a pair of expressions is equivalent, describe the properties of numbers and operations that justify the equivalence. If a pair is not equivalent, correct the mistake.

a. $2(x - 1)$ and $2x - 1$     b. $4(3 + 2x)$ and $12 + 8x$

c. $9 - (x + 7)$ and $16 - x$     d. $\dfrac{6x + 12}{6}$ and $x + 12$

e. $5x - 2 + 3x$ and $8x - 2$     f. $4x - x + 2$ and $6$

Computer algebra systems (CAS) have been programmed to use properties of numbers and operations like those described early in this investigation to expand, factor, and simplify algebraic expressions. To use a CAS for this purpose you need only enter the expression accurately and then apply the "expand" or "factor" commands from the CAS algebra menu.

You can also check equivalence of two given expressions by entering each as part of an equation and pressing the ENTER key. If the expressions are equivalent, the CAS will respond with "true."

**10** Compare the CAS output shown below with your answers to the following problems. Discuss and reconcile any differences.
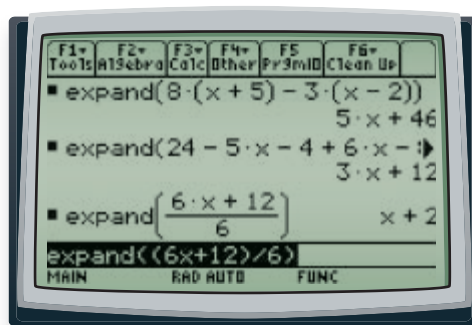


**a.** Problem 7 Part h

**b.** Problem 8 Part d

**c.** Problem 9 Part d

**11** Use the CAS that is available to you to perform the following algebraic procedures. Compare the CAS output with what you expect from your knowledge of algebraic manipulations and reconcile any differences.

**a.** What is the shortest expression equivalent to
$25x - 4(3x + 7) + 106$?

**b.** What expression equivalent to that in Part a is in simplest factored form?

**c.** What expression is equivalent in expanded form to
$5(7 - 3x) - (8x + 12)23$?

**d.** What results from asking your CAS to factor $a \cdot x + a \cdot c$?

**e.** What results from asking your CAS to factor
$a \cdot (x + b) - c \cdot (x + b)$?

**f.** What results from asking your CAS to expand
$a \cdot (x + b) - c \cdot (x + b)$?

# Summarize
## the Mathematics

In this investigation, you applied key properties of numbers and operations to evaluate the equivalence of expressions and to create equivalent expressions.

**a** Summarize these algebraic properties in your own words and give one example of how each property is used in writing equivalent expressions.

**b** How can you tell if expressions such as those in this investigation are in simplest form?

**c** What are some easy errors to make that may require careful attention when writing expressions in equivalent forms? How can you avoid making those errors?

***Be prepared to explain your thinking and examples to the class.***

## ✔ Check Your Understanding

For each of the following expressions:

**i.** Write two expressions equivalent to the original—one that is as short as possible.

**ii.** Describe the algebraic reasoning used to obtain each expression.

**iii.** Test the equivalence of each expression to the original by comparing tables and graphs.

**a.** $9x + 2 - x$

**b.** $3(x + 7) - 6$

**c.** $\dfrac{20 - 4(x - 1)}{2}$

**d.** $2(1 + 3x) - (5 - 6x)$

# *On Your Own*

## *Applications*

**1** To advertise a concert tour, the concert promoter paid an artist $2,500 to design a special collector's poster. The posters cost $2.50 apiece to print and package in a cardboard cylinder. They are to be sold for $7.95 apiece.

   **a.** Write expressions that show how to calculate cost, income, and profit if $n$ posters are printed and sold.

   **b.** Write two expressions that are different from, but equivalent to, the profit expression you wrote in Part a. Explain why you are sure they are equivalent.

**2** The video game industry is a big business around the world. Development of a new game might cost millions of dollars. Then to make and package each game disc will cost several more dollars per copy. Suppose the development cost for one game is $5,000,000; each disc costs $4.75 to make and package; and the wholesale price is set at $35.50 per disc.

   **a.** Write expressions that show how to calculate the cost of designing and making $n$ discs and the income earned from selling those $n$ discs.

   **b.** Write two different but equivalent expressions for profit from selling $n$ discs.

   **c.** Use evidence in tables, graphs, or properties of numbers and operations to justify equivalence of the two expressions from Part b.

**3** The historic Palace Theater offers students and seniors a $2 discount off the regular movie ticket price of $7.50. The theater has 900 seats and regularly sells out on the weekends. Marcia and Sam wrote the following expressions for income from ticket sales based on the number $x$ of discounted tickets sold for a sold-out show.

   **Marcia's expression:** $5.5x + 7.5(900 - x)$
   **Sam's expression:** $900(7.5) - 2x$

   **a.** Explain how Marcia and Sam may have reasoned in writing their expressions.

**b.** Use tables and graphs to check whether Marcia's and Sam's expressions are equivalent.

**c.** Write a simpler expression for income based on the number $x$ of discounted tickets sold for a sold-out show. Show how you could reason from Marcia's and Sam's expressions to this simpler expression.

**4** In art class, students are framing square mirrors with hand-painted ceramic tiles as shown at the right. Each tile is one inch by one inch.
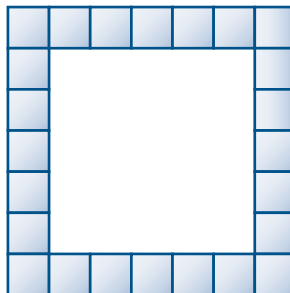
**a.** How many tiles are needed to frame a square mirror with side length 5 inches? 3 inches? 10 inches?

**b.** Write an expression for the number of tiles needed to frame a square mirror with side length $x$ inches.

**c.** One group of students came up with the following expressions for the number of tiles needed to frame a square mirror with side length $x$ inches. Explain the thinking that might have led to each of these expressions. Use tables, graphs, or algebraic reasoning to demonstrate the equivalence of the expressions.

   **i.** $(x + 1) + (x + 1) + (x + 1) + (x + 1)$

  **ii.** $(x + 2) + x + (x + 2) + x$

 **iii.** $4x + 4$

 **iv.** $4(x + 1)$

  **v.** $2(x + 2) + 2x$

**5** Are the following pairs of expressions equivalent? Explain your reasoning in each case.

**a.** $7 - 5(x + 4 - 3x)$ and $7 - 5x + 20 - 15x$

**b.** $7x - 12 + 3x - 8 + 9x - 5$ and $7x - 4 + 2x + 8 + 10x - 13$

**6** For each of the following expressions, write an equivalent expression that is as short as possible.

**a.** $3x + 5 + 8x$                 **b.** $7 + 3x + 12 + 9x$

**c.** $8(5 + 2x) - 36$           **d.** $2(5x + 6) + 3 + 4x$

**e.** $\dfrac{10x - 40}{5}$                  **f.** $5x + 7 - 3x + 12$

**g.** $3x + 7 - 4(3x - 6)$      **h.** $-7x + 13 + \dfrac{12x - 4}{4}$

**7** For each of the following expressions, combine like terms and then write in factored form.
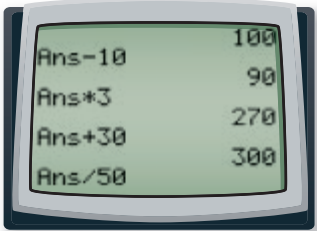
**a.** $6x + 5 + 9x$                **b.** $20 + 6x + 4 + 10x$

**c.** $32 + 20x$                   **d.** $13x + 6 - (2 - 3x)$

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## Connections

**8** One expression for predicting the median salary in dollars for working women since 1970 is $4{,}000 + 750(y - 1970)$, where $y$ is the year.

**a.** Write an equivalent expression in the form $a + by$. Explain how you know the new expression is equivalent to the original.

**b.** What do the numbers 4,000, 750, and the expression $(y - 1970)$ tell about the salary pattern?

**c.** What do the numbers $a$ and $b$ that you found in Part a tell about the salary pattern?

**9** Consider the following formula for transforming temperature in degrees Fahrenheit $F$ to temperature in degrees Celsius $C$.

$$C = \frac{5}{9}(F - 32)$$

**a.** Use the distributive property to rewrite the expression for temperature in degrees Celsius in the form $aF + b$.

**b.** Write a question that is more easily answered using the original expression for calculating the temperature in degrees Celsius. Write another question that is more easily answered using the expression from Part a. Explain why you think one expression is better for each question.

**10** Consider the following set of instructions.
Pick any number.
Multiply it by 2.
Subtract 10 from the result.
Multiply the result by 3.
Add 30 to the result.
Finally, divide by your original number.

```
                    100
Ans-10
                     90
Ans*3
                    270
Ans+30
                    300
Ans/50
```

**a.** Repeat the process several times with different starting numbers. What are your answers in each case?

**b.** Let $x$ represent the starting number. Write an expression showing the calculations for any value of $x$.

**c.** Write the expression from Part b in simplest equivalent form. Explain how it makes the results in Part a reasonable.

**11** The length of a rectangle is $\ell$ and the width is $w$.

**a.** Write at least three different expressions that show how to calculate the perimeter of the rectangle. Explain how you might reason from a drawing of a rectangle to help you write each expression.

**b.** Use the properties of numbers and operations, discussed in this lesson, to reason about the equivalence of the expressions you wrote in Part a.

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**12** Recall the formula $A = \frac{1}{2}bh$ for the area of a triangle where $b$ is the length of the base and $h$ is the height of the triangle. A trapezoid with bases of lengths $b_1$ and $b_2$ and height $h$ is shown below.
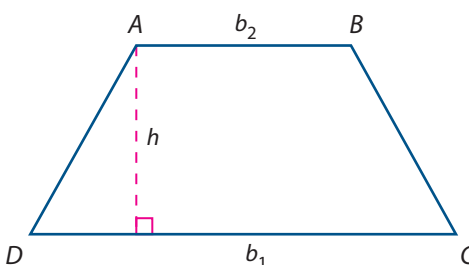
**a.** Make a copy of the diagram.

**b.** Draw $\triangle ACD$ and write an expression showing how to calculate its area.

**c.** Write an expression showing how to calculate the area of $\triangle ABC$.

**d.** Write an expression showing how to calculate the area of trapezoid *ABCD*.

**e.** Write a different but equivalent expression for the area of the trapezoid.

**13** Refer back to the Check Your Understanding problem of Investigation 1 (page 218). To study the profit prospects of different options in organizing a college basketball tournament, it might be helpful to construct a spreadsheet to calculate income, expenses, and profit that will result from various decisions.

**a.** Copy the expressions you developed for income, expenses, and profit in terms of the number of tickets sold $n$ based on the following information.

- Income is $60 per ticket sold, $75,000 from television and radio broadcast rights, and $5 per person from concession stand sales.

- Expenses are $200,000 for the colleges, $50,000 for rent of the arena and its staff, and a tax of $2.50 per ticket sold.

**b.** Use those expressions to complete the following spreadsheet so that you could explore effects of different ticket sale numbers. Enter the required numbers and formulas in column **B** of the spreadsheet.

| | A | B |
|---|---|---|
| 1 | Tickets Sold = | |
| 2 | Income = | |
| 3 | Expenses = | |
| 4 | Profit = | |

Spreadsheet sample.xls

**c.** Expand and modify the spreadsheet so that you could also adjust ticket price, average concession stand income, ticket tax, and television/radio broadcast rights to see immediately how profit changes. (*Hint:* Put the labels for those factors in cells of column **C** and then the values in adjacent cells of column **D**.)

## *Reflections*

**14** In Applications Task 2 about video game discs, any one of the following expressions could be used to calculate profit when $n$ discs are sold.

$$35.50n - 4.75n - 5,000,000$$
$$30.75n - 5,000,000$$
$$35.50n - (5,000,000 + 4.75n)$$

**a.** Explain how you can be sure that all three expressions are equivalent.

**b.** Which expression do you believe shows the business conditions in the best way? Explain the reasons for your choice.

**15** Think about a real situation involving a changing quantity. Write at least two different but equivalent expressions that show how to calculate the quantity.

**16** How do you prefer to check whether two expressions are equivalent: using tables of values, graphs, or algebraic reasoning? Why? Which gives the strongest evidence of equivalence?

**17** Each of the expressions in Applications Tasks 5–7 defines a linear function. However, none of those expressions looks exactly like $a + bx$, the familiar form of an expression for a linear function.

**a.** What features of expressions like those in the Applications tasks suggest that the graph of the function defined by that expression will be a line?

**b.** What might appear in an expression that would suggest that the graph of the corresponding function would not be a line? Give some examples and sketch the graphs of those examples.

**18** In transforming algebraic expressions to equivalent forms, it's easy to make some mistakes and use "illegal" moves. Given below are six pairs of algebraic expressions. Some are equivalent and some are not.

- Use tables, graphs, and/or algebraic reasoning to decide which pairs are actually equivalent and which involve errors in reasoning.

- In each case of equivalent expressions, describe algebraic reasoning that could be used to show the equivalence.

- In each case of an algebra mistake, spot the error in reasoning. Write an explanation that would help clear up the problem for a student who made the error.

**a.** Is $3(2x + 8)$ equivalent to $6x + 8$?

**b.** Is $4x - 6x$ equivalent to $2x$?

**c.** Is $8(2 - 6x)$ equivalent to $16 - 48x$?

**d.** Is $10 + 3x - 12$ equivalent to $3x + 2$?

**e.** Is $\frac{5x + 10}{5}$ equivalent to $x + 10$?

**f.** Is $-4(x - 3)$ equivalent to $-4x - 12$?

## *Extensions*

**19** Solve these equations. Show your work and check your solutions.

**a.** $3(x - 4) = 12$

**b.** $2(a + 1) = 8 + a$

**c.** $13 - (5 + n) = 6$

**CPMP-Tools**

**20** Look back at the statements of the Commutative and Associative Properties of Addition on page 220.

**a.** There are corresponding properties of multiplication. Write statements for the corresponding properties.

**b.** Give examples showing how the Commutative and Associative Properties of Multiplication can be used in writing equivalent algebraic expressions.

**21** The properties of numbers and operations discussed in this lesson can be applied to write equivalent expressions for any expression, not just linear expressions.

**a.** Consider the rule suggested for the income $I$ from the Five Star bungee jump as a function of ticket price $p$ from "Physics and Business at Five Star Amusement Park" in Unit 1.

$$I = p(50 - p)$$

Expand the expression $p(50 - p)$.

**b.** Consider the rule some students wrote for estimating the total population of Brazil *NEXT* year given the population *NOW*.

$$NEXT = 0.01 \cdot NOW + NOW - 0.009 \cdot NOW$$

Combine like terms to write a shorter expression on the right-hand side of the rule.

**22** Use the distributive property to write the expression $(x + 2)(x + 3)$ in expanded form. Is this a linear expression? Explain how you know.

**23** Write the following expressions in factored form.

**a.** $8x^2 + 12x$

**b.** $10x - 15x^2$

**c.** $3x^3 - 27x^2 + 18x$

**24** You have learned how a CAS can be used to expand or factor expressions, test the equivalence of two expressions, and solve equations. How could you solve a system of linear equations using a CAS? Test your ideas using the system below from Lesson 2 (page 199). Compare the solution with the solution you previously found.

**Surf City Business Center:** $y_1 = 3.95 + 0.05x$

**Byte to Eat Café:** $y_2 = 2 + 0.10x$

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*Review*

**25** Write an equation for the line satisfying the given conditions.

   **a.** passing through the point (0, 4) and having slope 3

   **b.** passing through the points (10, 7) and (20, 12)

   **c.** passing through the points (−3, 5) and (1, −3)

**26** Recall the Pythagorean Theorem which states that if $a$ and $b$ are the lengths of the legs of a right triangle and $c$ is the length of the hypotenuse, then $a^2 + b^2 = c^2$. There are many right triangles for which the hypotenuse has length 5. For example, if $a = 1$ and $c = 5$ then $1^2 + b^2 = 5^2$, or $b^2 = 24$. So, $b$ must be about 4.9 units long.

   **a.** Complete the table below by assigning different values to the leg length $a$ and calculating the length $b$ of the other leg.

| $a$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $b$ | 4.9 | | | |
| $c$ | 5 | 5 | 5 | 5 |

   **b.** Is the pattern of change in $b$ as $a$ increases a linear pattern? Explain.

**27** Consider the two functions $y = 2 + 0.25x$ and $y = -8 + 1.5x$. Use tables or graphs to estimate solutions for these equations and inequalities.

   **a.** $2 + 0.25x = 0$          **b.** $2 + 0.25x = -8 + 1.5x$

   **c.** $-8 + 1.5x > 0$         **d.** $2 + 0.25x \le -8 + 1.5x$

**28** Consider the quadrilaterals shown below. Assume that segments that look parallel are parallel, and that segments or angles that look congruent are congruent.

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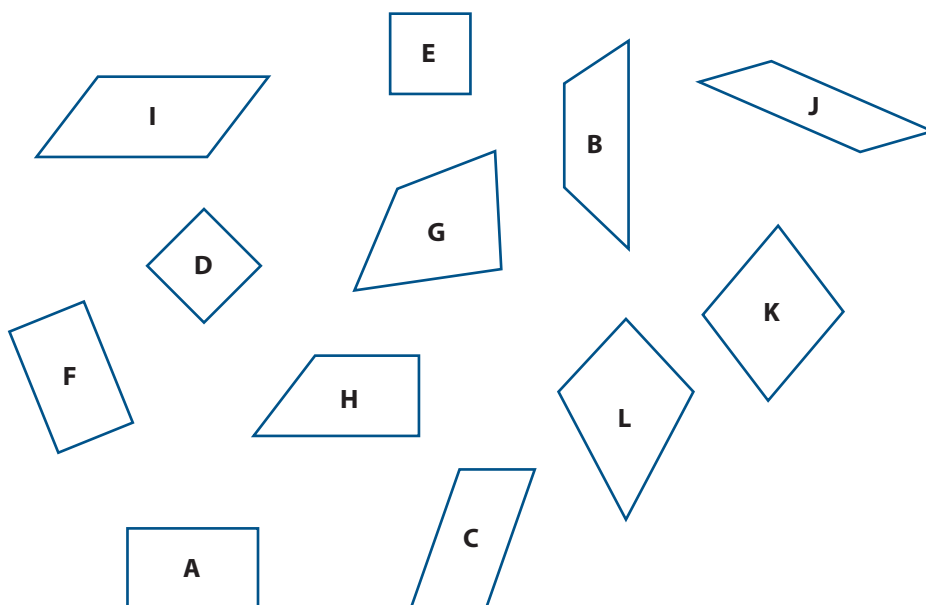
a. Identify all figures that are squares.

b. Identify all figures that are rectangles.

c. Identify all figures that are parallelograms.

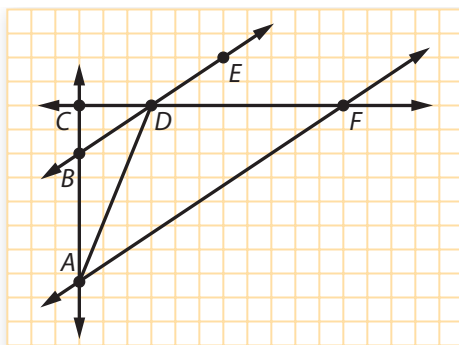d. Identify all figures that are trapezoids.

**29** The histogram below displays the number of books read over the summer by all of the ninth grade students at Treadwell High School.

**Summer Reading of Ninth Grade Students**
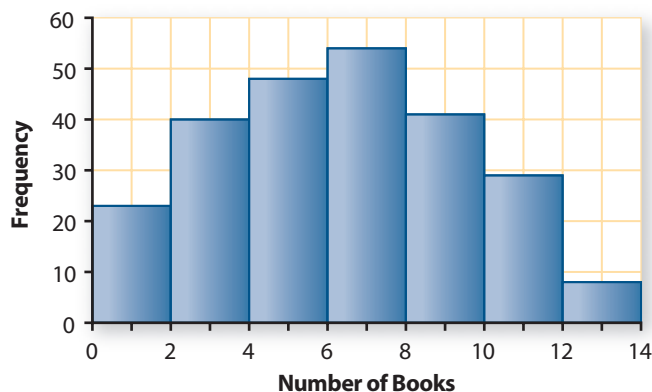


a. Emily estimated that the median of these data is about 8 books. Do you think she is correct? Explain your reasoning.

b. Will estimated that the mean of these data is about 6 books and the standard deviation is about 4 books. Do his estimates seem reasonable? Explain your reasoning.

**30** Using the diagram below, determine if each statement is true or false. In each case, explain your reasoning.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{DF}$

b. $m\angle FDA < m\angle CDA$

c. $\overleftrightarrow{BE} \parallel \overleftrightarrow{AF}$

d. $AD = DF$

# *Looking Back*

**T**hrough the lessons of this unit, you have seen that variables in many situations are related by linear functions. You have learned how to find rules that describe those functions. You also have learned how to use equations to answer questions about the variables and relations. This final lesson of the unit provides problems that will help you review, pull together, and apply your new knowledge.

**1** **Fuel Consumption** When private pilots make flight plans for their trips, they must estimate the amount of fuel required to reach their destination airport. When the plane is in flight, pilots watch to see how much fuel they have left in their tanks. The table below shows fuel remaining in the tanks at various times during a flight under constant speed for one type of small plane.

**Fuel Consumption**

| **Time in Flight** (in minutes) | 40 | 60 | 80 | 100 | 120 |
|---|---|---|---|---|---|
| **Fuel Remaining** (in gallons) | 50 | 45 | 40 | 35 | 30 |

a. Is fuel remaining a linear function of the time in flight? How do you know?

b. Determine the rate of change in fuel remaining as time in flight increases. What units describe this rate of change?

c. How much fuel was in the tanks at the start of the flight?

d. Write a rule that shows how to calculate the amount of fuel remaining in the *NEXT* hour given the amount of fuel remaining in gallons *NOW*.

e. Write a rule that shows how to calculate the amount of fuel remaining $F$ after $t$ hours in flight.

f. How much fuel remained in the tanks after 1.5 hours? After 3 hours?

g. At one point in the flight, the pilot observes that 5 gallons of fuel remain in the tanks. How much flying time is left?

**2** **Health and Nutrition** Even if we do not always eat what is best for us, most Americans can afford nutritious and varied diets. In many countries of the world, life is a constant struggle to find enough food. This struggle causes health problems such as reduced life expectancy and infant mortality.

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**a.** The data in the table below show how average daily food supply (in calories) is related to life expectancy (in years) and infant mortality rates (in deaths per 1,000 births) in a sample of countries in the western hemisphere. Make scatterplots of the (*daily calories, life expectancy*) and (*daily calories, infant mortality*) data.

**Health and Nutrition**

| Country | Daily Calories | Life Expectancy | Infant Mortality |
|---|---|---|---|
| Argentina | 3,136 | 74 | 20 |
| Bolivia | 2,170 | 64 | 56 |
| Dominican Republic | 2,316 | 67 | 36 |
| Haiti | 1,855 | 53 | 61 |
| Mexico | 3,137 | 73 | 28 |
| New Zealand | 3,405 | 78 | 6 |
| Paraguay | 2,485 | 71 | 37 |
| United States | 3,642 | 78 | 7 |

**Source:** *The New York Times 2003 Almanac.* New York, NY: Penguin Company, 2002.

Study the patterns in the table and the scatterplots. Then answer these questions.

  **i.** What seems to be the general relation between daily calories and life expectancy in the sample countries?

  **ii.** What seems to be the general relation between daily calories and infant mortality in the sample countries?

  **iii.** What factors other than daily calorie supply might affect life expectancy and infant mortality?

**b.** Economists use a linear model to try to predict the likely increase of life expectancy or decrease of infant mortality for various increases in food supply.

  **i.** Determine a linear regression model for calculating life expectancy from calories using the (*daily calories, life expectancy*) data pattern.

  **ii.** Determine a linear regression model for calculating infant mortality from calories using the (*daily calories, infant mortality*) data pattern.

**c.** What do the slopes of the graphs of your linear models say about the pattern relating life expectancy to daily calories in the sample countries? How about the relationship between infant mortality and daily calories?

**d.** Average daily calorie supply in Chile is 2,810. What life expectancy and infant mortality would you predict from the calorie data?

**e.** Brazil has a life expectancy of 68 years.

  **i.** For what daily calorie supply would you predict this life expectancy?

**ii.** The actual daily calorie supply for Brazil is 2,938 calories. What does the difference between your predicted value and the actual value tell about the usefulness of the model you have found?

**f.** What life expectancy does your model predict for a daily calorie supply of 5,000? How close to that prediction would you expect the actual life expectancy to be in a country with a daily calorie supply of 5,000?

**③ Popcorn Sales** Many people who go to movies like to have popcorn to munch on during the show. But movie theater popcorn is often expensive. The manager of a local theater wondered how much more she might sell if the price were lower. She also wondered whether such a reduced price would actually bring in more popcorn income.
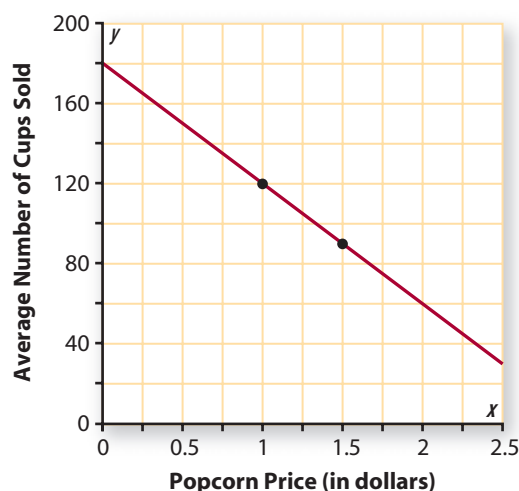
One week she set the price for popcorn at $1.00 per cup and sold an average of 120 cups per night. The next week she set the price at $1.50 per cup and sold an average of 90 cups per night. She used that information to graph a linear model to predict number of cups sold at other possible prices.

**Popcorn Sales**



**a.** Write a rule for the linear model. Explain what the slope and $y$-intercept of the model tell about the prospective number of popcorn cups sold at various prices.

**b.** Write and solve equations or inequalities to answer the following questions.

   **i.** At what price does your model predict average daily sales of about 150 cups of popcorn?

   **ii.** At what price does your model predict average daily sales of fewer than 60 cups of popcorn?

   **iii.** How many cups of popcorn does your model predict will be sold per day at a price of $1.80 per cup?

**c.** Use the rule relating average daily number of cups sold to price to make a table relating price to income from popcorn sales. Explain what the pattern in the table tells about the relation between price and income.

**④** Solve the following equations and inequalities. Use each of the following methods of solving—table, graph, and algebraic reasoning— at least twice. Check your answers.

**a.** $9 + 6x = 24$           **b.** $286 = 7p + 69$

**c.** $6 - 4x \leq 34$           **d.** $8 + 1.1x = -25$

**e.** $20 = 3 + 5(x - 1)$        **f.** $17y - 34 = 8y - 16$

**g.** $1.5x + 8 \leq 3 + 2x$        **h.** $14 + 3k > 27 - 10k$

**⑤** **Party Planning** The ninth grade class at Freedom High School traditionally has an end-of-year dance party. The class officers researched costs for the dance and came up with these items to consider.

**Party Planning**

| Item | Cost |
|---|---|
| DJ for the dance | $350 |
| Food | $3.75 per student |
| Drinks | $1.50 per student |
| Custodians, Security | $225 |

The question is whether the class treasury has enough money to pay for the dance or whether they will have to sell tickets.

a. Which of the following function rules correctly express dance cost $C$ as a function of the number of students $N$ who plan to come to the dance? Explain your reasoning.

$C = 350 + 3.75N + 1.50N + 225$
$C = 5.25N + 575$
$C = 575 + 5.25N$
$C = 580.25N$

b. Write and solve an equation or inequality to determine how many students could come to the dance without a ticket charge if the class treasury has $950.

c. Write and solve an equation or inequality to determine how many students could come to the dance with a ticket charge of only $2 if the class treasury has $950.

**⑥** Using algebraic expressions to help make sense out of problem situations is an important part of mathematics. Writing expressions and function rules is often a first step. Being able to recognize and generate equivalent algebraic expressions is another important skill.

a. Write rules for the linear functions with graphs passing through the indicated points.

  **i.** $(0, -3)$ and $(4, 1)$
  **ii.** $(0, 3)$ and $(6, 0)$
  **iii.** $(2, -6)$ and $(8, 12)$

b. Write rules for the linear functions with graphs having the given slopes and passing through the given points.

  **i.** slope $= 3$; passes through $(4, 12)$
  **ii.** slope $= \frac{2}{3}$; passes through $(-6, -1)$
  **iii.** slope $= -4$; passes through $(17, 82)$

Lawrence Migdale

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**c.** Compare the following pairs of linear expressions. Are they equivalent? Explain your reasoning in each case.

  **i.** $4.2x + 6$ and $(1 - 0.7x)6$

  **ii.** $4C - 3(C + 2)$ and $-6 + C$

  **iii.** $0.3S - 0.4S + 2$ and $\dfrac{20 - S}{10}$

**7** Solve the following system of equations using calculator- or computer-based methods and by algebraic reasoning.

$$\begin{cases} y = 35 + 0.2x \\ y = 85 + 0.7x \end{cases}$$

# Summarize
## the Mathematics

**Linear functions can be recognized in graphs, tables, and symbolic rules, or in verbal descriptions.**

**a** Describe how you can tell whether two variables are related by a linear function by looking at:

  **i.** a scatterplot  **ii.** a table of values

  **iii.** the form of the function rule  **iv.** a description of the situation

**Linear functions often describe relationships between an input variable $x$ and an output variable $y$.**

**b** Write a general rule for a linear function. What do the parts of the rule tell you about the function it represents?

**c** Explain how to find a value of $y$ corresponding to a given value of $x$, using:

  **i.** a graph  **ii.** a table  **iii.** a symbolic rule

**d** Explain how to determine the rate of change in $y$ as $x$ increases, using:

  **i.** a graph  **ii.** a table  **iii.** a symbolic rule

**e** Explain how you can solve a linear equation or inequality using:

  **i.** a graph  **ii.** a table  **iii.** algebraic reasoning

**f** Explain how you can solve a system of linear equations using:

  **i.** a graph  **ii.** a table  **iii.** algebraic reasoning

*Be prepared to share your descriptions and explanations with the whole class.*

## ✔ Check Your Understanding

Write, in outline form, a summary of the important mathematical concepts and methods developed in this unit. Organize your summary so that it can be used as a quick reference in future units and courses.